THE THEORY OF INTERNATIONAL VALUES.

I.

International trade meaning in plain English trade between nations, it is not surprising that the term should mean something else in Political Economy. In technical usage international is distinguished from home trade by the existence of barriers which prevent owners of the means of production in one region—or, more generally, sphere of industry—from employing those means in another sphere. Or is it easier to say that home trade is distinguished from international by the tendency to equal remuneration of efforts and sacrifices: to an equality of profits, and an equation of the net advantages in different occupations? The general conditions which determine equilibrium are the same for both species of trade; the only difference is that in the case of the home trade there are one or two more equations.

Such is I think the essential attribute of the term international trade as used by theoretical economists; the properties of geographical and political separation, though usually understood, are not those from which the principal conclusions flow.

The flexibility of this definition escapes from the objection that there is no difference in the present age between international and domestic trade. Let it be granted that capital and perhaps business power is free to flow to all parts of the earth. Yet labour cannot be conceived as flowing so freely. The world is not yet in the condition of the American colonies where, if Virginia damnified Maryland by a tax, it is said that the immobility of industrial agents, as Professor Bastable says, in his admirable discussion of the definition in question.—International Trade, ch. 1.

The plan of putting international before domestic trade—treating it as the rule rather than as an exception—may have historical as well as theoretical justification, if we agree with Professor Bastable that 'the first exchanges were international (or rather intertribal).’ Commerce of Nations, p. 7.

Business power at least, if not labour, has in several cases been transferred from England to foreign countries, in order to avoid hostile tariffs. See Diplomatic and Consular Reports, Spain 1893, C 6885, 112, p. 18. I have heard of other instances consequent on the McKinley Tariff.
habitant of Maryland would transfer himself to Virginia. Presumably there may be a considerable difference in the level of advantage in different countries before labour flows from one to another. Suppose, however, that the conditions of international trade proper ceased to exist, there would still remain the quasi-international trade between the parties to Distribution. There would still be a great gulf between employers and employed across which work is transported in exchange for finished products.

According to this view the fundamental principle of international trade is that general theory which Jevons called the Theory of Exchange, and Prof. Marshall describes as 'an inquiry into the balancing of the forces of Demand and Supply,' which constitutes 'the kernel' of most of the chief problems of economics. It is a corollary of the general theory that all the parties to a bargain look to gain by it. Foreign trade would not go on unless it seemed less costly to each of the parties to it to obtain imports in exchange for exports than to produce them at home. This is the generalised statement of the principle of Comparative Cost, with respect to its positive part at least. The negative clause, that the value of articles in the international market is not proportioned to the cost—the 'efforts and sacrifice'—incurred by the respective producers, is superfluous, if the definition here proposed is adopted. Why should there be any correspondence between cost and value in the absence of the conditions, proper to domestic trade, on which that equality depends?

In a complete treatise on international trade it would be proper to dwell at length both on the general principle and the corollary; on the one hand contemplating the tendency towards maximum satisfaction, which constitutes the grandest generalisation of Economics; and on the other hand applying the doctrine of Comparative Cost to explain the peculiarities of existing commerce—why such and such articles are exported from one country and imported to another.

1 Quarterly Journal of Economics, October 1892.
3 Principles, Book V. ch. III.
4 The principle is employed by almost all mathematical writers on economics, among whom Professor Marshall may be distinguished as stating carefully the limitations, under the existing social régime, of the 'doctrine of maximum satisfaction' (Principles of Economics, Book V. ch. 12, §7); and Dr. Irving Fisher as appreciating the mysterious analogies between the maximum principles in physics and in human affairs. 'Mathematical Investigations in the Theory of Value and Prices,' From Transactions of the Connecticut Academy, Vol. IX., July 1892.
5 As Professor Taussig has done in his brilliant article on 'Aspects of the Tariff Question,' in the Quarterly Journal of Economics for 1889, p. 291.
THE THEORY OF INTERNATIONAL VALUES

But it is proposed to confine this study to those portions of the theory which have at once some bearing on practice, and also a high degree of generality. I shall endeavour, in a first article, to express in as simple language as possible some propositions of this double character. A mathematical version of the same propositions will form the second part. The third part will contain a critical review of the principal writers on international trade.

Of the propositions relating to international trade which are at once general and bear on practice the most important, I think, are those which attribute advantage or detriment—whether for one nation or several—to changes in the supply of, or demand for, articles of trade. Such are the answers to the questions: Would a tax or a bounty, an improvement or deterioration in the means of communication, abundance or scarcity of an exported article, be beneficial to the home country, or to all parties? The answers to such questions vary with the data, which require to be carefully distinguished.

One distinction—which indeed hardly needs to be pointed out, since it is the similarity, not the difference, which generally escapes notice—is that which has been already indicated between international trade proper, relating to separated regions, and the analogues thereof which may be termed quasi-international trade. Another distinction, which one might have a priori supposed to be very obvious, is between the interests of the home country and that of the world at large. Yet, strange to say, a confusion between ideas so different as part and whole pervades many of the arguments in favour of Free Trade; the complaints of List 1 against ‘the School’—the followers of Adam Smith—on this ground are too well founded. 2 The equivocation might be compared to that which it was reserved for Prof. Sidgwick to point out in the term Utilitarianism—referring sometimes to the Greatest Happiness of the individual, and sometimes to that of the whole.

1 National System.

2 The amiable confusion between one’s own or one’s country’s exclusive advantage and that of the world at large may be attributed to Mr. Gladstone, when he asks—in his article on ‘Free Trade or Protection,’ in the North American Review, Vol. cl.—‘why, if Protection is a good thing, it should not be adopted by the United States in their internal trade.’

Even the most clearheaded of writers, James Mill (Elements of Political Economy, ch. III. § 16, p. 159, ed. 1821) and Professor Bastable (International Trade, p. 123, and ‘Incidence and Effects of Import and Export Duties,’ in the Report of the British Association for 1889, p. 6 of the essay, p. 446 of the Report), seem not to distinguish very sharply the ideas of advantage to the world and to a particular nation.
Another important distinction is between small and large changes; the characteristic of the latter being such an alteration in the scale of production that the law of increasing returns is brought into operation [or the converse alteration]. Thus the 'improvement' in the process of manufacture of an exported article considered by Mill in his great chapter (Bk. III. ch. 18 §. 5) is presumably of the order 'small'; the change contemplated by him in an earlier section (§ 2), from a time 'when each country produced both commodities to an established trade,' may well be—but is not necessarily—large. Another distinction to which it is proper to call attention is between an impediment to trade [or an improvement] in general and that particular kind of obstruction [or encouragement] which a tax [or bounty] constitutes. The proceeds which may accrue from a tax form an item which is sometimes left out of account in the balance of advantages.1

Other principles of classification requiring no comment are the distinction between changes originating in the home country, or abroad; between those affecting primarily exports, or imports; between the case of two countries, and that of several countries; and so forth.

It will be sufficient here to select the most instructive cases; requesting the reader to attend carefully to the issue, and to stay condemnation, until appeal has been made to the tribunal of mathematical reasoning.

The simplest case is where the question is whether the advantage of the home country is increased by an increase in the supply of foreign articles, in the sense that the foreigner is willing to give a greater quantity of those articles in exchange for any the same quantity of native produce, the increase being supposed to be on a small scale.2 Upon the general principle that a cheap market is advantageous to the buyer, the home country is benefited; whatever the cause of the increased supply, whether it is due to an improvement in the production of the foreign articles, or a greater desire on the part of the foreigners for the produce of the home country, or ceteris paribus an increase in their numbers. Conversely

1 Thus the project of a differential tax on foreign produce (in favour of the colonies) is described by an eminent free-trader as a demand that 'England should tax herself to the amount of 104 millions'; as if England would be a loser to that extent. In the view which I adopt the amount received by the Government is to be set against the amount paid by the people.

a diminution in the supply of foreign goods is detrimental to the home country.

The technical use of the term increase of supply must here be kept in mind. It is quite possible that the home country might suffer by the foreign customer becoming better supplied with commodities in general. It is well observed by Mr. Medley, an ardent free-trader, that the adoption of free trade by all nations—which of course, according to him, implies the increase of their wealth—might prove detrimental to England. The poverty of the foreigner may quite conceivably be advantageous to the native.

Suppose a new country exchanging with an old one food for highly manufactured products. An increased deficiency in necessaries on the part of the old country, or of a large section thereof, always supposing—perhaps an imaginary supposition—that their efficiency is not thereby impaired, rendering them more eager for the supplies derived from the new country, is apt to benefit the new country considered as a whole. However, the particular section of the home country which supplies services analogous to those of the foreigner—considered as an isolated group—may well be prejudiced by the poverty of foreign labour.

This last consideration suggests a fresh topic—international competition; which may however be subordinated to the present one (the change in the supply of foreign goods) by observing that when a competitor with the home country deals with the foreigner, the ‘supply’ of foreign goods is diminished. Formal reasoning and common sense concur in regarding such competition as an evil to the home country.

The solution is not so simple when we consider changes originating on the side of the home country. Such changes may be divided into two classes, according as they originate on the side of supply, or demand: exports, or imports. Under the former

1 Fair Trade Unmasked.
2 Ceteris paribas, of course: not supposing that, when the real remuneration of the foreign labourers is diminished, that of his employer is increased; as Mill and Cairnes do in effect; when, discussing the effect on international values of low wages in a foreign country, they use wages in the peculiar Ricardian sense (Pol. Econ., Book III. ch. 25, § 4 and Leading Principles). These passages will be discussed in our Part III.
3 Professor Walker in his powerful and impartial article on ‘Protection and Protectionists’ in the Quarterly Journal of Economics for April 1890, admits it to be quite possible that in some branches of American industry ‘the manufacturers pay higher wages for a given quantity of labour than are paid abroad.’
4 See Part II. Mill’s paradoxically low estimate of this evil will be considered in Part III.
head the simplest case is where there has occurred an improvement [or the reverse], a diminution [or increase], in the cost of production of an exported article; the case considered by Mill in the fifth section of his great chapter on International Values. As may be gathered from Mill's reasoning, the improvement may prove detrimental to the exporting country.\(^1\) It is true that Mill obscures the subject by taking as the measure of the gain of trade the alteration in the rate of exchange between exports and imports rather than the truer measure of advantage which the principles of Consumers' and Producers' Rent afford. However, a representative case may be put which brings out the implication latent in Mill's reasoning. It will be recollected that Mill supposes an improvement in the production of linen which Germany exchanges for cloth imported from England; in which case he shows it to be a possibility that 'Germany will obtain cloth on more unfavourable terms and at a higher exchange value than before' \(\text{loc. cit. \S 5, par. 6}\). Now suppose that the same amount of productive forces are expended on linen by the German manufacturer before as after the improvement. If the increase in productivity has been ten per cent., where before there were 100 units of linen produced, there are now 110 units produced. But if the demand for linen be increased 'in a less proportion than the cheapness,' whereas the German used to receive, say, 100 units of cloth, he will now receive less than 100. For an equal outlay in the way of cost he receives a less return. Whence it follows, if we make the further supposition that linen is not an article of German consumption, that the exporting country is damnified by the improvement; and by parity of reasoning may be benefited by a restriction of its exports. It is clear that the data which have been supposed may be considerably modified without the conclusion being destroyed.

But indeed, without invoking Mill's stupendous chapter, the proposition is sufficiently supported by common sense. It is a commonplace that a bad harvest is good for farmers in the absence of foreign competition. As Ricardo says, 'if we lived in one of Mr. Owen's parallelograms and enjoyed all our productions in common, then no one could suffer in consequence of abundance; but as long as society is constituted as it now is, abundance will often be injurious to producers, and scarcity beneficial to them.' \(^2\)

\(^1\) This view and some others here adopted seem to differ from those of an eminent living economist, whose writings on International Trade will be noticed in the third part of this study.

\(^2\) Protection to Agriculture, \S 4, sub fin.
Let us assume, according to Gregory King's law,¹ that a deficiency in quantity by a tenth may raise the value of the harvest by three-tenths. Now, suppose that the harvest has been an average one; but that, as the grain is sent to market, a tenth leaks out, or is intercepted by robbers (to use a favourite free trade metaphor). The total value will be, as before, raised; so beneficent (to one party) may be the effect of what Cherbuliez calls artificial dearth.²

An example of an impediment to export, other than a tax accruing to the exporting country, is a transit duty levied on the exports from one country to another by a third party. It is conceivable that the native states of India might be benefited by the duty which we levy on opium passing through our territory, if China had no other means of satisfying her demand for opium.

A similar effect might be produced by an increase in the cost of transporting the exported article from the locality of its production to the port, supposing that there is no corresponding drag on importation.³

The effect of a variation in the cost of transport generally will be compounded of different tendencies: since an impediment on exportation and on importation in general affects both countries, so far as each both exports and (in return voyages) imports. Since, out of the four tendencies thus compounded, one only (variation in the cost of exportation by natives)—and that one only on certain conditions—would lead to a benefit for the natives from an aggravation of the cost of transport, it may be presumed that in general such an aggravation is very unlikely to be advantageous to the home country.

The case of an improvement in the process of manufacture⁴ of an article which is both exported and consumed at home, is also a compound between the certain gain to the native consumer and the possible loss to the home country in the way of foreign trade. It is quite possible that the latter tendency may prevail over the former, just as in the case of farmers⁵ who may gain more as producers, than they lose as consumers, by a bad harvest.

An instructive example of the principle under consideration

² Dictionnaire d'Économie politique. Art. 'Disette.' Cf. Art. 'Abondance,' by Bastiat.
³ As might well occur in a round-about trade.
⁴ Mill, Pol. Econ., ch. 18, § 5.
⁵ Above, p. 40.
is afforded by the question whether a diminution of the output of the home country's exports consequent upon a limitation of working hours is necessarily injurious to the country. That this question is to be answered in the negative is well argued by Mr. Sidney Webb in his article on 'Limitation of the Hours of Labour' in the Contemporary Review for December 1889.\(^1\) It is noticeable that the advocate of socialistic measures dwells on propositions relating to the trade between two nations; he does not bring on the scene a third country competing with the socialistic one. An advocate on the other side would probably represent the whole argument as vitiated by this omission. The judicial position is intermediate between these two. If the demand of the foreign customer for our goods, prior to, or abstracted from, the existence of a competing country, is such as to render a restriction of exports advantageous to the home country, it may still be possible, notwithstanding the existence of competition, to obtain that sort of advantage though in a less degree. As Professor Marshall says with reference to this question, 'the influence of foreign trade competition in this connection can be proved to be different from what it at first sight appears.'\(^2\)

It should not be conceived, I think, that the conditions favourable the successful restriction of exports are altogether exceptional. Mill, after distinguishing three varieties of conditions inquires 'which is the more probable,' and decides in favour of that variety which, as we have already seen, is favourable to the policy of restriction.\(^3\) Accordingly, if each nation could only deal with one other, either of the pair might often play the game of restriction with advantage. But no doubt the existence of competition modifies the foreigner's law of demand for the native articles in such wise \(^4\) as to render that game much less gainful.

It is to be observed that the advantage which has been described results from a drag on exports which need not be a tax. \textit{A fortiori} of course when the impediment is a tax accruing to the exporting country. The latter proposition is much more generally accepted, I think, than the former.\(^5\) It is often stated with the unnecessary limitation that the home country

---

\(^1\) See p. 878, Vol. LVI.
\(^3\) \textit{Pol. Econ.}, Book III, ch. 18, § 5 last par.
\(^4\) Cf. \textit{Anis}, p. 99.
\(^5\) The latter is explicitly admitted even by McCulloch; the former not even by Mill.
must have an absolute monopoly of the exporting article. That she should furnish a considerable portion of the total supply might suffice.

Coming next to changes originated on the side of imports (to the home country), let us consider a restriction on importation such as a transit duty imposed by a third power on imports into the home country. Such an impediment on imports, unlike one on exports, is never advantageous to the home country. The duty levied by the Indian Government on opium transported through Bombay from the Native States might conceivably benefit those States, but not the Chinese.

A tax indeed on imports the proceeds of which accrue to the home country may be beneficial to that country: but not in so many cases, not with as great probability, as a tax on exports. The positive part of this statement is proved by Mill; but the negative part is less easy to establish by the purely literary method.

A sense of this difference between the effect of a tax on exports and one on imports is perhaps traceable in the division of opinion with regard to the question whether a tax on imports can fall on the foreigner—a division of opinion greater than exists with regard to the corresponding question concerning exports. That a tax on imports may prove a net gain to the home country is admitted by the Chalmers, but it is denied by the common free-trader and even by competent economists when expressing themselves carelessly. It may be as well to adduce instances of these contrary judgments; so that my argument in favour of the proposition in question may appear neither paradoxical nor otiose.

In favour of the proposition the following high authorities may be cited:—Mill (Political Economy, bk. v. ch. iv. § 6).

'A tax on imported commodities almost always falls in part upon the foreigners.' . . . 'Those are in the right who maintain that taxes on imports are partly paid by foreigners.'

Senior. (Outlines, 184).

'A part of the taxes received by the Government of one country is often paid by the inhabitants of another.'

---

1 E.g. Rogers, Six Centuries, p. 79, 'there must be no other source of supply.'
2 For the evidence of this asymmetry I must refer to the forthcoming Part II.
3 Pol. Econ., Book V. ch. 4, § 6, passages quoted below, p. 47.
4 McCulloch, for instance, admits the latter, but denies the former (Principles of Pol. Econ., Part II., ch. v., sub finesm).
Seligman (*Incidence of Taxation*, ch v.).

'Ve will be seen how erroneous is the doctrine of those extremists who maintain that the loss to the consumer is measured by the proceeds of the import duties.' . . . 'The price of Sumatra tobacco has risen by only a fraction of the tax.'

Compare the admissions made by Professor Bastable in his paper on 'Incidence and Effects of Taxation' so often referred to, and Professor Nicholson's reasoning in his masterly paper on 'Tariffs and International Commerce.'

On the other side Mongredien (*Pleas for Protection Examined*):

'Import duties on foreign goods fall on the consumers of the importing country and are paid by them.'

Sydney Buxton (*A.B.C. of Free Trade*):

'Duties on goods are paid for by the people who consume those goods, and not by the people who produce them.'

Sir J. Lubbock at the Congress of the Chambers of Commerce of the Empire, 1892, says, 'I maintain the proposition that the duties are paid by the consumer.' (*Chamber of Commerce Journal*, July, 1892, *Supplement*, p. 28.)

The opinion is not confined to Free-Traders. Mr. McKinley (*North American Review*, cl. p. 742) writes—

'If the duty is put on the non-competing foreign products, the consumer in the United States will pay every dollar of that tax.'

An instructive statement of the common free trade opinion is found in Mr. Strachey's singularly brilliant report on the effect of the German tariff (Parl. Papers, 1884-5, LXXXI). Mr. Strachey speaks of

'The axiom of political economy that a tax on foreign commodities is borne by the importing country. No one could so much as state [the contrary] without exposing himself to the charge of having no sense of humour.'

No one certainly will bring this charge against Mr. Strachey; for his report is probably the Wittiest blue-book in existence; one of the wisest too, if we except this particular passage. Mr. Strachey seems to himself to have proved his case when he has demonstrated—by some very interesting statistics—that the price of the taxed

---

1 *Scottish Geographical Magazine*, September, 1891.
article in the importing country exceeds its price in the exporting
country by just the amount of the tax, abstracting cost of trans-
port. But quis dubitavit? If, as is or was recently the case, there
is a tax of two dollars per ton on hay imported from Canada into
the United States, the cost of transport being here insignificant,
the price per ton on the American side of the frontier will be
two dollars higher than on the Canadian side. The question is
whether it is the American price that has gone up, or the Canadian
price which has gone down. The latter happens to be the case.¹

A similar ignoratio elenchi is committed by a still higher
authority, Roscher, when he argues that Germany must pay the
full amount of the tax which she imposed on wheat imported
from America; for that the price in Germany (account being
taken of cost of transport) exceeds that in England by exactly
the amount of the tax.² But how does he know that the imposition
of the tax did not cause America to offer her wheat to England
on better terms than before? It may be the American price
which has gone down, not the German price which has
gone up.³

Probably the highest authority and weightiest argument in
favour of the proposition in question are those of M'Culloch, who
holds⁴ that the project [of obliging foreigners to contribute to the
revenue of the nation]⁵ is wholly imaginary, and that duties on
imports are always paid by the importers, and never by the ex-
porters⁶; the reason being that the exporters must obtain the
rate of profits prevailing in their country, and therefore cannot
after the tax lower the price which before the tax only just
afforded the ordinary profits.⁷

Let us examine this reason.

¹ As shown in the Report of the Subcommittee of the Committee of Finance
(Senate U.S.) by Senator Merrill (Rep. 788). Here are some extracts from the evidence:
'The duty of five cents per dozen imposed upon eggs by the McKinley tariff is paid
by the foreign producer, not by the consumer.' . . . 'They have dropped the valuation
on most farm products just about the amount of the duty imposed by the McKinley
bill.' . . . 'No question they have to take 30 per cent. less for their horses.'
Mr. Edward Atkinson in his comments on this Report (Taxation and Work, ch.
xxv.), after ridiculing the 'delusion that one of the effects of a duty imposed in
this country upon a given import is to depress the price of that article in the country
in which it is produced, and that by such reduction the burden of our tax is put
upon that country' (p. 190) admits (p. 194), that 'our duties upon the products of
Canada have unquestionably had that effect.'
² Finanzwissenschaft, p. 411, Note 4. ³ Cf. Bastable, Incidence, p. 3.
⁴ Principles of Political Economy, Part I., ch. v., sub finem. Cf. Taxation and
Funding, Part II., ch. v.
⁵ M'Culloch's argument is employed by Mongredien (Plead for Protection) and
other extreme Free-traders.
First, as pointed out by Prof. Bastable,\(^1\) price may be lowered without profits being diminished, if the cost of production varies with the margin. Thus a tax imposed by the United States on certain kinds of agricultural produce imported from Canada\(^2\) might result in the diminution of the quantity, the cost of production, and the price of that produce. This idea of a freely sliding margin is indeed highly theoretical, but so is the objector’s idea of equal profits in all occupations.

More important in practice, if less familiar in theory, is the analogous case in which the burden falls—not on rent proper—but on ‘quasi-rent.’ Suppose an import tax laid on tin plates. The tax might be paid out of the surplus gains of the more successful foreign manufacturers,\(^3\) while the less successful would be driven out of the field.

No doubt if the tax imposed were a very heavy one, such as is now fashionable, say 50 or a 100 per cent., it is not to be expected that the foreign exporters should lower their price to that extent. The price of tin plates then will rise in the home country. Accordingly a net loss corresponding to that rise of price appears to be inflicted on the home country. But it appears so only while we confine our attention to immediate effects. When an engine pushes against a carriage the immediate effect is that the buffer of the carriage is pressed back. When the buffer has been pressed back to a certain point the carriage begins to move, and the buffer of the next carriage, and in fine the whole train. The propagated influence of a tax may be similar, in a case where the demand of the foreigner for the products of the home country—say food and raw materials—is very urgent. The export of tin plates being checked, the foreigners find a difficulty in paying for the imports which they so much require. To restore the equation of international trade they are constrained to offer their exports other than tin plates—exports in general—on terms less favourable to themselves. It is quite conceivable that the gain which the home country derives from this readjustment of trade may exceed the loss which it derives from the rise of the value of tin plates. As Mill says in his splendid and candid section on Protectionism: ‘A country which prohibits some foreign commodities does, \textit{ceteris paribus}, obtain those which it does not prohibit at a less price than it would otherwise have to pay.’

\(^1\) \textit{Incidence} p. 3, \textit{Cf. International Trade} p. 45. See also Sidgwick, \textit{Pol. Econ.}, Book III. ch. v. § 3.
\(^2\) Above, p. 45.
An import tax in the case supposed would resemble the export tax before considered, in tending to check the exports from the home country. For a country so circumstanced it might be disadvantageous to 'grow more cotton and cereals,' as Mr. Gladstone recommends the Americans. How should the native labour, which but for the check to exports would have been employed in producing them, be now most advantageously employed? Quite possibly on 'tin plates'; thereby rendering the native demand for foreign goods less pressing, and thus more fully satisfying the conditions which must exist in order that the foreigner may be taxed.

These arguments are not affected, or rather become *a fortiori*, by the existence of 'invisible' exports or imports of the nature of capital lent, or interest paid. For by the operations which have been described the value of money will have been increased in the foreign country and decreased in the home country. Accordingly the natives as lenders or debtors will now have to give less of their own produce, and as borrowers or creditors will receive more of the foreigner's produce.

It has been shown that under conceivable circumstances advantage may result to the home country from a tax on exports or imports. But will it result under given circumstances? A negative answer, I think, may be given in some concrete cases; in many 'the only answer is that an answer is impossible'; as Professor J. S. Nicholson demonstrates in his essay on 'Tariffs and International Commerce.' The affirmative answer is described by him as 'part of the casuistry of economics,' like the discussions of moral philosophers concerning the occasional justification of mendacity. 'Free trade, like honesty, still remains the best policy.'

This analogy seems singularly just to one who agrees with Mill as a moralist that 'even this rule [truth], sacred as it is, admits of possible exception'... that 'the exception ought to be recognised, and, if possible, its limits defined'; and with Mill as an economist, that in particular cases 'taxes on imports

2 In his article on 'Free Trade and Protection,' in the *North American Review*. See Mr. Blaine's criticism of his advice. Ibid.
4 In the *Scottish Geographical Magazine* for September, 1891.
5 *Utilitarianism*, ch. I.
are partly paid by foreigners.'

‘England will gain at the expense of Germany not only the whole amount of the duty but more’ by an export tax.

Bounties being ‘negative taxes,’ as Cournot says, it might have been expected that in cases where a tax is detrimental, a bounty would be beneficial. It is not so however; a bounty, whether on exports or imports, takes more from the Government than it gives to the public; so long as we confine our attention to changes which are not organic in the sense already explained.

But when we consider large changes apt to be attended with a reorganisation of trade, many of the preceding propositions no longer hold good. An increased supply, a greater cheapness of foreign goods, may now, I think, prove disadvantageous. A bounty may prove advantageous upon principles indicated by Prof. Marshall, by calling into play the law of increasing returns. Upon similar principles, a tax on imports may foster native industries, it may be advantageous in its ulterior as well as its more immediate effects; in the way of protection, as well as in the way of what may be called in a large sense revenue.

I hope it may be allowable to define my subject so as to exclude a detailed examination of the free-trade controversy. On the general issue I have nothing to add to what I have learnt from the first-rate writers who have treated of the subject, in particular Mill, and Prof. Sidgwick, and Prof. Marshall. As I read, protection might procure economic advantage in certain cases, if there was a Government wise enough to discriminate those cases, and strong enough to confine itself to them; but this condition is very unlikely to be fulfilled.

---

1 Book V. ch. IV. § 6.  
2 Ibid.  
3 Ibid.  
4 Of course I agree with Mill and living writers that for one nation to benefit itself at the expense of a greater loss to others is contrary to the highest morality, which takes the greatest happiness of all as its end. ‘The justice . . . of destroying one of two gains in order to engross a rather larger share of the other does not require discussion’ (Mill. Book V. ch. X. § 1). But, in an abstract study upon the motion of projectiles in vacuo, I do not think it necessary to enlarge upon the horrors of war.  
5 Once more I can only offer a proleptic reference to my Part II.  
6 Principles of Economics, Book V. ch. 12.  
7 Including producers' and consumers' rent, as well as the receipts of the Treasury.  
So far we have been regarding exclusively the advantage of the home country. When we take in the interest of all parties we are met with the axiom that any interference with exchange diminishes the sum total of advantage resulting to all parties concerned. The axiom, like most of the propositions with which we are concerned, presents two aspects according as we consider small or organic changes. With reference to the former case it may be accepted without qualification, except so far as the level of utility, so to speak, is regarded as different in different countries; the exports of one country as compared with another costing more labour, and the imports exciting more satisfaction.

When we consider large changes, developing new industries, it is conceivable, as Prof. Sidgwick has argued that an interference with the 'natural' course of international trade may be beneficial to all parties.

Much of what has been hitherto said refers primarily to the case of trade between two countries. But the transition to the more general case is easy. As Mill says, 'trade among any number of countries must take place on the same essential principles as trade between two countries . . . Introducing a greater number of agents precisely similar cannot change the law of their action' (Political Economy, Book III., ch. xviii., § 3).

The preceding propositions relate especially to international trade proper. But many of them may be transferred to that quasi-international trade of which the principal example is the transaction by which the national produce is divided between the owners of the agents of production. The principal characteristic peculiar to international trade proper is, I think, the possibility of a nation benefiting itself by a tax on exports and imports. There may indeed be a tax on the transactions between 'nations' in the generalised sense—such as a tax on wages—but the proceeds of the tax would accrue to the community, not to one of the groups.

It is useful, I think, to contemplate the theory of distribution as analogous to that of international trade proper. It is seen, for instance, that the intention which seems to inspire some of the leaders of labour to raise wages by restricting the supply of

---

1 Compare Professor Marshall, Principles, Book III. ch. VI. § 2, par. 9.
3 It will be recollected that the competition of a third country was treated as affecting the demand of one of the two countries; above p. 38.
labour is *prima facie* quite consistent with general principles. But a doubt may occur whether the special conditions are favourable for carrying such a policy to any great length; when the transaction between the *entrepreneur* and the workman, who supplies an agent of production in return for a share of the produce, is likened to that sort of international trade which England used to have with the Southern States of America, when she imported materials (cotton) and exported the finished article.

Again it is instructive to regard the transaction between landlord and farmer as a sort of international trade. The familiar proposition that 'rent does not enter into price,' or into cost of production, may thus be seen in a clearer light. But this is one of the topics which may better be treated in the mathematical part which is to follow.

F. Y. Edgeworth
THEORY OF INTERNATIONAL VALUES

II

The mathematical version of the theory consists either of Geometry or Algebra.

Geometry is directly applicable only to the simplest possible cases. If more than two commodities are considered, solid geometry must be called in. The dimensions of space are not adequate to represent the case of more than three variables.

Geometry therefore might appear to have no application to reality; since countries importing or exporting only one article exist only in imagination. But the geometrical representation of this imaginary case is useful as suggesting theorems which may be seen to admit of extension to more concrete cases.

The simplest geometrical representation of international trade appears to be a construction first used by Professor Marshall and explained by him in the mathematical appendix to his Principles.2

In Figure 1, the curve O E, which might be called England's Supply-and-Demand curve, signifies that for a certain quantity O x of English produce, say 'cloth,' exported, the quantity O y of German produce is demanded. The supply of linen and demand for cloth on the part of Germany are similarly expressed by the curve O G.

With respect to these curves it is not, I think, necessary to make the supposition which is usually made with respect to more familiar demand or supply curves—namely, that while the rate of exchange represented by the curves is varied, the rate of exchange between one of the ordinates and all other articles—the price of all other articles, as it would usually be expressed—remains constant.3 Rather a movement along a supply-and-demand curve of international trade should be considered as attended with

1 See the first article published in the Economic Journal for March 1894; hereinafter referred to briefly as ante.
2 Note 12, second edition.
The theory of comparative costs is not very prominent from the mathematical point of view. It may be represented geometrically as follows. Let the cost of production at first be supposed constant; then the terms on which England could have obtained linen in the absence of the trade may be represented by a straight line $OS$, if \( \tan SOX = \text{ratio of the cost of production of a unit of linen to that of a unit of cloth} \). In order that England may obtain linen cheaper with than she could without the trade, the point of equilibrium must be above the line $OS$. It must be below the line $OT$, in order that Germany may be benefited. To generalise this theory there should be substituted for the straight line $OS$ (and mutatis mutandis for $OT$) a curve of constant utility as the import.

---

1 As by Messrs. Auspitz and Lieben when they take money of constant marginal utility as the import.
2 As by J. S. Mill.
advantage, or 'indifference-curve' (not shown in the figure), representing states for which the advantage to England is no greater than if there had been no trade. That the point of equilibrium falls between the respective indifference-curves is the geometrical version of Comparative Costs. The expression which occurs in some of the best writers, that international value 'depends on' comparative cost, is seen from this point of view to be a very loose expression.

In investigating the incidents attending differences in the conditions of supply and demand it is important to distinguish the varieties of data. This purpose may be assisted by the following logical tree, or ramification; where the capital letter corresponds to a positive, the small Roman to a negative attribute.

A, International trade proper; a, quasi-international trade (in particular, distribution).
B, the case of two nations only; b, of several.
C, where we regard the interest of only one, our own, country; c, where we regard the interest of all parties concerned.
D, where we regard present advantage only; d, future also.
E, where we are concerned only with functions of the simple form proper to 'short periods' (such as the curves in Fig. 1), and accordingly the changes contemplated are in a sense small; e, where more complicated functions and organic changes are considered.
F, where the change considered originates in a foreign country; f, in the home country.
G, an improvement or impediment other than a bounty or tax; g, a bounty or tax.
H, where the change originates on the side of supply: such as increased facility of producing or exporting native commodities; h, on the side of demand: such as an increased desire for, or facility in admitting, foreign commodities.

By ringing the changes on these positive and negative attributes some hundreds of different cases can be distinguished; thus (1) A B C D E F G H, (2) A B C D E F G h, (3) A B C D E F g H, (4) A B C D E F g h; and so on up to 26.

1 See the present writer's Mathematical Psychics, pp. 31-29.
2 No doubt, as Professor Bastable has pointed out, when there are numerous competing nations, the limits fixed by the principle of Comparative Cost are much narrowed; and accordingly it becomes less incorrect to regard the principle as sufficient to determine international value.
3 As proposed ante, p. 37, par. 2.
4 Marshall, Principles of Economics.
5 Ante, p. 88.
6 Described below, p. 436.
But of the compartments thus formed many would be empty, such as those which combine c, regard for the interest of all nations, with F or f, distinguishing natives and foreigners. It is proposed to consider only the more important cases—namely, those which have been summarily treated in the preceding article.

A B C D E F. — This is the case of international trade proper, between two countries, regard being had to the interests of the home country only, and immediate or direct effects only being considered; and a certain simplicity in the law of demand and supply for both countries being assumed, a change is supposed to occur in the terms on which the foreigner is willing to trade.

The increase of the supply of foreign produce (in the sense that more of it is offered at each rate of exchange) is represented in Fig. 2 by the displacement of the foreign curve O G to O G'. Whatever the direction of the native or the foreign curve in the neighbourhood of their intersection, it will be found that in every case the new intersection has travelled along the native curve away from the origin. Whence the change is bene-

---

1 Ante, p. 38, last par.
2 Consistent with the condition that the equilibrium should be stable.
ficial to the native country. Conversely, a diminution in the
offer of foreign goods is prejudicial to the home country; as
may be seen by taking the dotted curve as the original one.

A B C D E f G H.—The case of f, a change originating in
the home country, is not so simple. The answer varies
according as the letters, after f, are capital or lower case,
designate positive or negative attributes. In each case much
turns upon what Mill calls the extensibility of demand. This
property may be thus contemplated. Draw a line parallel to the

axis Y touching the curve O E in T (Fig. 3). Divide this line into
a number of equal small parts: Tr1, r1 r2 below T, and T s1 s2
above T. Each interval corresponds to an increment in the
value of X with respect to z, that is the number of units of z
given in exchange for a unit of H. Join r1, r2, &c., s1, s2, &c.
to O; and from the points r1, r2, &c., s1, s2, &c., let fall per-
pendiculars—not shown in the figure—on the axis Y. Then
it appears that below the point T an increment in value of

1 If this proposition is not self-evident, I may refer for a proof of it to my
Mathematical Psychics, p. 115.
2 Ante, p. 39, last par.
3 Book III. ch. 18, s. 1.
X corresponds to a more than proportionate increase in the quantity of Y demanded; and conversely, above the point T. We may describe the curve above T as elastic, below inelastic. Each of the cases comprised under A B C D E f are divisible into four subcases, according as the native or foreign curve is elastic or inelastic.

A B C D E f G H I.—This is the case of a decrease (or increase) in the supply of exports due to a cause other than the imposition (or remission) of a tax: such as a change in the cost of production, or transport. The four subcases are represented by the four varieties of Fig. 4; O E being as before the native curve, and O E’ what it becomes by the change considered.

Subcase (1) is where both native and foreign curves are elastic. The native curve O E becomes transformed by the impediment to O E’. In the new equilibrium indicated by the point Q, R Q of X is given in exchange for Q S of Y. But Q cannot be a position of greater advantage than P’, where the horizontal through Q cuts the original curve. For, on the most favourable supposition that the impediment affects only exportation, not production for internal consumption, England’s offer in exchange for O R would be reduced by the impediment from O S’ to O S, so that Q would be a position of just equal advantage as P’. But P’ is a position of less advantage than P (being nearer the origin as you move along the curve). Thus the native country is prejudiced by the change.

The converse variety of the subcase, where an improvement, not an impediment, has supervened, may be investigated by treating O E’ as the original, O E as the displaced curve. Whence it appears that the native country is advantaged by the change.

In subcase (2), where the native curve is elastic, the foreign inelastic, by a parity of reasoning the natives may be benefited by an impediment, and prejudiced by an improvement.4

In subcase (3), where the native curve is inelastic, the foreign elastic, the natives are prejudiced by an impediment and benefited by an improvement, as in subcase (1).

In subcase (4), where both curves are inelastic, the natives may be benefited by an impediment and damaged by an improvement, as in subcase (2).

These results may be summed up in the diagram forming Fig. 5, which shows the consequences of an impediment;

2 Ante, pp. 40, 41.
3 For instance, a transit duty imposed by a third country. Ante, p. 41, par. 2.
4 Ante, p. 41, penultimate par.
the symbol + denoting advantage to the natives *oc teris paribus*, or abstracting the effects on internal trade; the symbol − denoting disadvantage without qualification. To exhibit the consequences of an improvement converse signs should be used.

![Diagram](image){#fig5}

_A B C D E F G h._—In the case of an impediment affecting imports, the displaced curve is formed by lengthening the ordinate instead of shortening the abscissa of the primary curve. Where the native curve is elastic, that is in subcases (1) and (2), the same figures will serve for h as for H. But in subcases (3) and (4) the diagrams forming Fig. 6 must be substituted for those which are proper to case H. As in case H, *mutatis mutandis*, the consequence of an impediment may be represented by the change from P to P'; those of an improvement from P' to P (see Fig. 6).

The diagram on p. 432 (Fig. 7), corresponding to that on this page, shows the consequences of an impediment to importation. The consequences of an improvement are obtained by changing each of the signs. It will be observed that a restriction on exports is not so certain to be prejudicial to the country imposing it as one on imports.

_A B C D E f g H._—The case of a tax *differs from that of an*

---

1 *Ante,* p. 42, last par.
impediment in that the change is not now from P to P', but from P to Q. To consider whether this change is advantageous or not we may employ the conception of an *indifference-curve* or locus of positions of trade which are of equal advantage as any
assigned position \( P \). \( P \) being on the supply-and-demand curve \( O \text{E} \), it may be shown that the indifference-curve touches the vector from the origin to that point, \( O \text{P} \) in Fig. 8.

Let the native indifference-curve through \( P \) cut the foreign demand-curve \( O \text{G} \) in \( M \). Then, if \( Q \), the new position of equilibrium, on the curve \( O \text{G} \) (see Fig. 8), is above \( M \), inside the indifference-curve, as in Fig. 8, the natives are benefited; if \( Q \) is below \( M \) the natives are prejudiced. In the subcase illustrated by Fig. 8, viz. subcase (1), it is in general uncertain whether \( Q \) is above or below \( M \). The consequence represented by the sign – in the case of an impediment (Fig. 5) becomes now +. But the sign + (in subcases (2) and (4) ) becomes à fortiori +.

These consequences of a tax on exports are exhibited in the annexed diagram, Fig. 9.

![Diagram](image)

From the reversibility of the positive and negative cases, which has so far prevailed, it might have been expected perhaps that a bounty should be advantageous in cases where a tax is detrimental. But this analogy is misleading; so long, at least, as we confine ourselves to attribute \( E \). ² The action of a bounty may be represented by considering in Fig. 4, \( Q \) as the original, and \( P \) the displaced point. In every subcase it will be found that the

1 Mathematical Psychics, p. 21.
2 Ante, p. 48, par. 2.
displaced point is on the **right** of the line joining the origin to Q; that is, **outside** the indifference-curve which touches the line at Q on the **other side.**

(See Fig. 8.)

A B C D E f g h.—The consequences of a tax on imports, investigated by parity of reasoning, are represented in the diagram which forms Fig. 10.

![Diagram](image)

Comparing this diagram with the preceding, we infer that a nation is more likely to benefit itself by an export than an import tax.

The want of symmetry between the effects of restrictions, and in particular taxes, on exports and imports, is perhaps the conclusion which can be most peculiarly and exclusively attributed to the mathematical method. The truth does not seem to have been clearly attained by the ordinary methods.

A B C e.—So far we have supposed the curves O E and O G to be of the simple form shown in Fig. 1. In considering complicated re-entrant forms like that in Fig. 11, it will be convenient to begin by restoring the usual supposition that the marginal utility of one of the commodities is constant. Thus let us for a moment

---

1 This conclusion agrees with that of Messrs. Auspitz and Lieben, as was to be expected, their curves being of our Class E. (See *Theorie der Preise*, p. 425.)

2 *Ante*, p. 43.
regard O E as a supply-curve indicating that a certain quantity of cloth O x is supplied in exchange for a certain quantity of a commodity whose marginal utility may be regarded as constant, say money. Then two kinds of supply-curve may be distinguished, (I) representing the amount of cloth which would be offered at each price, no account being taken of the change in the offer due to the alteration in the scale of production for different values of the primary supply-curve, as we may call it. It seems to be much the same as Professor Marshall's short period supply-curve. (II) Next let us take account of the change in the offer due to alteration in the scale of production; and so form a series of primaries corresponding to each value of x: Mr. Cunynghame's 'successive cost-curves.' If now at each point on the abscissa an ordinate is erected, the locus of intersection with the corresponding 'successive cost'-curve forms a secondary supply-curve: Mr. Cunynghame's supply-curve; and, as I understand, Professor Marshall's 'long-period' supply-curve.

It is a nice question whether a primary cost-curve can be regarded as re-entrant in the manner represented in Fig. 11. Perhaps we may with sufficient generality consider that it cannot. The secondary curves are (a) sometimes of the simpler form represented in the earlier figures; (b) sometimes re-entrant as in Fig. 11. Curves of the former kind, that is IIa, have many properties in common with species I; in particular that movement along the curve in a direction from the origin is attended with advantage.

We have just been regarding O E as a supply-curve. Now let us regard it as a demand-curve in this sense that O y linen (see Fig. 1) is demanded in exchange for O x of a commodity whose marginal utility is constant, say money. Then from this point

---

1 As in Messrs. Amsps and Lieben's constructions.
2 Econ. Journ., vol. ii.
3 As argued by the present writer elsewhere (Address to Section F of the British Association Report, 1889, Note J). Though at a given rate there may be several maxima of advantage, there can be only one position of greatest possible advantage. Since, then, the motive of the economic man is greatest possible, rather than merely maximum advantage, it should seem that the ordinate of the supply-curve corresponding to each value of tan P O X must be unique; discontinuous for the individual who must be conceived as jumping from one branch to another when a certain value of tan is reached, but continuous for the community since the point of transition will be different for different individuals. On the other hand, there may exist friction obstructing the movement from a small to a large scale of production; and so two branches of the curve exist simultaneously. In this case, as pointed out by the present writer (Mathematical Psychics, Appendix 7), the tract between T and T'—points where tangents drawn from the origin touch the curve—is not a genuine demand-and-supply curve, being a locus of minimum advantage.
of view also, if the law of demand is considered to vary with the scale of consumption, as Mr. Cunynghame supposes, the curve may prove to be re-entrant.\footnote{Ascending in Mr. Cunynghame's construction. See Econ. J., vol. ii.} I submit, however, that this cause of abnormality is less important and less capable of being formulated than the influence of the scale of production on cost.

Not that from either point of view an exact determination of the curve is to be expected; we must be content with general descriptions: such as elastic and inelastic, re-entrant or not. Still less definiteness is attainable when, combining the two views which have just been distinguished, we restore our original view of the demand-and-supply curve \(OE\): as representing the interchange of two articles of variable marginal utility.

The consequences of the property of re-entrance may be considered under the head (d), which indeed is with difficulty separated from (e); since, in fact, organic changes only occur in long periods.

A B C d.—Many of the propositions, stated under preceding heads, no longer hold when we consider organic changes extending over long periods. Thus it ceases to be universally

\[\text{Fig. 11.}\]
true that an increase in the supply of foreign commodities is beneficial to the native country.\(^1\) For the curve O\(G\) being shifted upwards might strike the native curve in the neighbourhood T (Fig. 11), corresponding to a lower value of the native produce with respect to the foreign,\(^2\) and a lower value of the native goods may be attended with detriment to the native country.\(^3\)

Again a bounty ceases to be universally disadvantageous. For, in the manner shown by Professor Marshall with respect to a different construction, a bounty may shift the point of equilibrium to a position more advantageous to the community.

A B c D E. — When we consider the interest of both parties, not of one only, the chance of benefit resulting from interferences with trade is diminished. The presumption that any such interference impairs the total utility is well illustrated by Messrs. Auspitz and Lieben, on the tacit assumption that what may be called the hedonic worth of money is the same in both countries. The generalised form of that assumption — appropriate to our system of co-ordinates, which does not represent money — is that if for each party a curve be drawn cutting at right angles the system of indifference-curves — called by the present writer a preference-curve — the same distance along such a curve corresponds to the same increase or diminution of advantage on both sides. This is no doubt an allowable assumption, in the absence of knowledge to the contrary. But, when we know that one party is much better off than another,\(^4\) the assumption may be illegitimate.\(^5\)

A B c d e. — The doctrine that interferences with trade are detrimental to the community of nations becomes more questionable when we consider organic changes operating for a consider-

\(^1\) Above, p. 427.

\(^2\) The proposition set forth in the books (e.g. Mill, Political Economy, Book III., ch. xviii.) that the setting up of trade is advantageous to both countries assumes that the curves [or the analogous algebraic functions in the general case] with which we have to deal are of the form I or II (a). In that case the position of stable equilibrium may be regarded as a point of maximum of advantage in excess of the adjacent minimum formed by the position of null trade, viz. the origin. But, if curves of the form II b prevail, then a position of stable equilibrium, though a maximum, may be attended with less advantage than the position of null trade.

\(^3\) Aust, p. 48.

\(^4\) This is most likely to occur, I think, in the quasi-international trade between the parties to Distribution.

\(^5\) This, if not already evident, may be contemplated by regarding the contract-curve as the locus of points (Mathematical Psychics, p. 31 et seq.) at which the preference-curves of the two parties coincide with opposite directions. According to the assumption in question, it would be indifferent, from the point of view of the general good, whether all the advantage of trade accrued to one party, or both had a share.
able time. The possibility that such measures should be attended with advantage to all is well shown by Professor Sidgwick in his chapter on protection.¹

A b C D E F.—The case of trade between several nations which lends itself best to geometrical illustration is that of a third party competing with the home country, as we may call that one whose advantage is exclusively regarded, for trade with foreigners.

In Fig. 12 let O G be the foreign curve, O e the native, O e'

the competing, and O E compounded of the last two. The detriment inflicted on the home country by the competition may be described as the change in a backward direction along the curve O e from the intersection of O e with O G to p, where the line O P cuts O e.

It is to be observed that competition does not necessarily deprive a country of the advantage which it may derive by a restriction of imports. For suppose that in the absence of competition the conditions described in subcase (2) of A B C

¹ Ante, p. 49, par. 2.
existed, so that the home country could benefit itself by a
restriction of imports; then after the rise of competition it may
still be possible for the home country to benefit in the way
described under the heading referred to. The restriction will
transform O e to O e', O E to O E', P to P', p to q, which is apt to
be a position of equal advantage as p', and therefore of greater
advantage than p. It may be observed that this species of benefit
to the home country may be made possible by competition, not
having been so before, if O G is inelastic at its intersection with
O E, but not at its intersection with o e.
a.—The incidents of quasi-international trade—e.g. between
the parties to Distribution—do not lend themselves to geometry so
good as to algebra, on which we now enter.

In entering upon the more complicated part of the subject, it
is well to recall Professor Marshall's warning words: 'When a
great many symbols have to be used, they become very laborious
to any one but the writer himself,' and 'it seems doubtful
whether any one spends his time well in reading lengthy transla-
tions of economic doctrines into mathematics, that have not been
made by himself.' It is easier to strike out a new path for oneself
than to plant one's steps in the footprints of another.

It is almost sufficient to indicate the general scope of the
inquiry—namely, to determine that state of trade for which
the sum of the utilities of all parties concerned regarded as
functions of the amounts of commodity consumed, less the
sum of the disutilities regarded as functions of the amount pro-
duced, is a maximum; subject to the conditions that what is
bought is sold, what is consumed is produced, the 'law of indiffer-
ence,' the existence of non-competing groups, and so forth.

A few more particular directions may be added.

Let us begin with the case next in point of simplicity to that
which has been treated: where there are two countries, one of
which exports two articles, the other one article. Let x and y
be the amounts of the two articles exported by the first country,
and z the amount of the article exported by the second country.
Let X and Y be the amounts of the articles produced in the first
country which are consumed in that country; and Z the amount
of the article produced in the second country which is consumed
in that country.

1 Ante, p. 42.
3 Cf. Marshall, Principles, 2nd edition, note xii.; and the formulae given
by the present writer in the notes to the Address to Section F of the British
Association (1889).
Let us first consider the abstract case in which the cost of production is constant; say $a_1, a_2$ units of work\(^1\) in the first country go to a unit of each of its two products respectively; $b_1$ units of work in the second country to a unit of its product. Suppose also at first the number of units of work available to be a fixed quantity, say $A$ and $B$, in the two countries respectively. Then we have

$$\begin{align*}
1. & \quad a_1(x + X) + a_2(y + Y) = A \\
2. & \quad b_1(z + Z) = B.
\end{align*}$$

The advantage of the first country which is to be maximised, subject to the first of the above-written conditions, and the corresponding advantage of the second country, may be written—

$$\begin{align*}
1. & \quad \Phi(X, Y, z) \\
2. & \quad \Psi(z, y, Z).
\end{align*}$$

The position of equilibrium is determined by the values of the variables which make each of the above-written expressions a maximum; subject to the conditions stated by equations (1), and to the further condition—

$$3. \quad a_1x + a_2y = vb_1z;$$

where $v$ is the rate of exchange between the product of work in the two countries, the number of units of work in the first country, of which the product is equivalent to the product of a unit of work in the second country.

That $\Phi$ and $\Psi$ should each be a maximum, subject to equation (1), may be expressed by proposing each of the following expressions to be maximised—

$$\begin{align*}
1. & \quad \Phi(X, Y, z) - \lambda [a_1(x + X) + a_2(y + Y) - A] \\
2. & \quad \Psi(x, y, Z) - \mu [b_1(z + Z) - B],
\end{align*}$$

where $\lambda$ and $\mu$ are indeterminate factors.

The expressions (4) become by equation (3)—

$$\begin{align*}
1. & \quad \Phi(X, Y, z) - \lambda [a_1x + a_2y + vb_1z - A] \\
2. & \quad \Psi(x, y, Z) - \mu \left[\frac{1}{v} (a, x + a_2y) + b, Z - B\right].
\end{align*}$$

Differentiating the first of these expressions with respect to $X, Y, z$ respectively, and the second of the expressions with respect to $x, y, Z$ respectively, we have six equations, which with the pair of equations (1) and the equation (3) make nine equations, to determine the nine unknown quantities, $x, y, z, X, Y, Z, \lambda, \mu, v.$

\(^1\) More exactly 'effort and sacrifice'; involving at least two dimensions of disutility, labour and waiting.
Eliminating the last six of these variables, we obtain three equations of the form—

$$\begin{align*}
\Phi_1 (x, y, z) &= 0 \\
\Psi_1 (x, y, z) &= 0 \\
\Psi_2 (x, y, z) &= 0;
\end{align*}$$

which are the analogues of the demand (and supply) curves proper to the case of two commodities; e.g., \(\Phi_1\), giving the amount of imports demanded by the first country in exchange for assigned amounts of export, \(y\) and \(z\). The position of equilibrium may be regarded as the intersection of the three surfaces designated by equation (6).

Conclusions analogous to those which have been obtained for the case of two commodities are easily discerned to be obtainable in the case of three or more variables. Thus, if the second country has an urgent demand for one of the commodities, say \(x\), of the first country, it is possible that an export tax on \(x\) may be beneficial to the first country; while an export tax on \(y\) might not have that effect.\(^1\)

I do not know that any fresh conclusions are presented by the case of many variables. Accordingly it may be left to the reader to elaborate that case.\(^2\) It will be sufficient here to indicate how some of the concrete circumstances which have been abstracted may be restored.

First, the cost of production may be treated as varying with the amount produced by regarding \(a, x [a_2, x, \&c]\) not as the product of \(x\) by a constant, but as a definite function of \(x\).

Again, the amount of work available may be treated as variable, by regarding \(A [B, \&c.]\) not as a constant, but as a function of disutility, which disutility is to be subtracted from \(\Phi [\Psi, \&c.]\) in order to obtain the expression which is to be maximised.\(^3\)

The cost of transport may be introduced by regarding the littoral of one country as the scene of the market, and treating

\(^1\) Cf. above, p. 484.

\(^2\) A statement of the general case is given by Professor Pareto in his able article on 'Teoria Matematica dei Cambi Forestieri,' in the Giornale degli Economisti, 1894, Art. 9 et seq.

It may be observed that the formulae given by Professor Pareto (in the earlier part of his article), after Professor Walras, as proper to the case of industrial competition (domestic trade), are also applicable to that case of trade between 'nations (or non-competing groups') in which each commodity is produced by only one nation. The formulae do not express the essential attribute of domestic trade, viz. the tendency to equality in the net advantages of different occupations. Such net advantages, being of the nature of total utility, could not be expressed by formulae involving only final utility. This is the gist of my criticism of Professor Walras, to which Professor Pareto replies in the article referred to (loc. cit. p. 144).

the cost of importing foreign articles thereto as part of the cost of production.¹

It is unnecessary to show how the number of commodities and number of countries may be multiplied. What Mill says of the theory of value in general is particularly applicable to the mathematical version of it: ‘The further adaptation of the theory ... may be left with great advantage to the intelligent reader.’

It remains only to acknowledge my obligation to Professor Marshall’s unpublished chapters on foreign trade. These are the chapters alluded to in the Preface to the Principles of Economics as having been printed for private circulation and sent to many economists. Part of their substance is contained in the first volume of the Principles; part may be looked for in the second volume. What is written on the subject after a perusal of the privately circulated chapters, and pending the publication of the second volume, can make no claim to originality or permanence—like the light of the planet which precedes the rising of the sun, borrowed from and destined to be effaced by the prime orb.

F. Y. Edgeworth

[To be continued.]

THEORY OF INTERNATIONAL VALUES

III

The theories stated in two preceding articles are now to be sustained by, or maintained against, the authority of the principal writers on the subject. They may be divided into two classes, (I.) English and (II.) Continental; a division almost coincident with that between those who have not, and those who have employed mathematical methods.

I. (1) Ricardo.—Foremost in the first class is the founder of the theory,

Quo nihil majus generatur ipso,
Nec viget quidquam simile aut secundum.

The incomparable vigour of Ricardo's chapter on foreign trade has not been approached by any of his successors. The main propositions of the theory—the principle of comparative cost (M'Culloch's edition, p. 77), the change in the quantities and prices of commodities consequent upon foreign trade (p. 73, cf. p. 80 sub finem), the difference in the value of money in different countries (p. 79 et sqq.), are stated by Ricardo more briefly, and perhaps more clearly, than by J. S. Mill. Mill seems to have the advantage only in one respect; his recognition of the case in which an impediment to trade may be beneficial—or an improvement prejudicial—to one of the countries. It may be observed that the circumstance on which this property depends, the demand in the other country being 'increased in a greater proportion than the cheapness,' to use Mill's phrase (Pol. Econ. xviii. § 5), did not escape Ricardo (p. 73, par. 2).

The only scruples which the chapter may excite are removed by recollecting Ricardo's peculiar phraseology: the sense in which he employs the terms 'value,' and 'wages' or 'real wages,' and

1 See Econ. Journal, 1894, March and September.
3 Cf. Ricardo, p. 82, par. 2.
his elliptical use of *either* capital or labour where we might expect *both*. These explanations apply to the following passages:—

We should have no greater value if, by the discovery of new markets, we obtained double the quantity of foreign goods in exchange for a given quantity of ours (p. 72).

The country may have 'greater skill' and 'better machinery' used in the manufacture of exportable commodities; yet 'the rate of profits will probably differ but little'; wages, or the real reward of the labourer, may be the same in both (p. 81).

If capital freely flowed towards those countries where it could be most profitably employed, there could be no difference in the rate of profit, and no other difference in the real or labour price of commodities than the additional quantity of labour required to convey them to the various markets where they were to be sold' (p. 77).

(2) *J. S. Mill.*—Mill's contributions to the subject are contained in his stupendous chapter on *International Values* (Pol. Econ. Book III. ch. xviii.), the chapters on the Distribution of the Precious Metals, and the Competition of different Countries in the same Market (ibid. chs. xxi. xxv.) and the sections treating of the effects produced on international exchange by duties on exports and imports (Book V. ch. iv. § 6), and the Doctrine of Protection to Native Industry (Book V. ch. x. § 1); and the corresponding passages in the Unsettled Questions.

Mill's exposition of the general theory is still unsurpassed. He presents clearly all the leading features: the distinction between international and home trade (Bk. III. ch. 2, last par.), the former requiring us to 'fall back upon an antecedent law, that of supply and demand (ibid. ch. xviii. § 1);' the sense of 'cost' in which 'a country gets a commodity cheaper when it obtains a greater quantity of the commodity with the same expenditure of labour and capital' (ibid. § 9); the peculiarity that international values are not 'in the ratio' (ibid. and cf. ch. xvi. §1) of cost in that sense; but that a variation of cost in that sense will be attended with a variation—though not in general an equal variation—in international value (Book III. ch. xviii. §5). The additions and corrections which Mill's work has received will be noticed in the course of the following more detailed review.

Mill begins by considering the establishment of a trade between two nations. His classical illustration—the exchange of English cloth for German linen—has been much imitated, but little improved. The opening of a trade, which is considered

---

1. *Cf.* Book III. ch. xvi. §1. The term 'anterior' in this passage, of which Jevons complains (*Theory*, p. 215, 2nd ed.) fits well that conception of the distinction which has been adopted in this study (see Part I. par. 1).
in the first four sections of the great chapter, being a change of the kind which we have designated as simple or continuous does not differ essentially from the facilitation of (an already established) trade which is considered in the fifth section. The latter case may indeed be regarded as the more general since it comprehends both the case in which the facilitation is beneficial to both countries, the case to which the opening of trade presumably belongs, and also the case in which the facilitation is prejudicial to one party.

Mill is, I think, the first—indeed almost the only—economist who has stated the latter proposition. The statement would have been more complete if he had explicitly affirmed the converse proposition that an impediment to trade may be beneficial to one party.

It would have been well too if Mill in his chapters on International Values, and on the Competition of Different Countries (Book III. chs. xviii., xxv.), had treated the cost of production in each country not as constant, but as varying with the quantity produced—as his successors have done. The deficiency however is partly made up in the chapter on Taxes on Commodities (Book V.), where, with special reference to international trade, it is pointed out that 'duties on the produce of land or of mines might be so high as to diminish materially the demand for the produce, and compel the abandonment of some of the inferior qualities of land or mines. Supposing this to be the effect, the consumers, both in the country itself and in those which dealt with it, would obtain the produce at smaller cost' (§ 6).

It is a more serious complaint that Mill takes as the measure of the advantage which a country derives from trade, the increase in the international value of its exports. He thus confounds

1 *ante*, pp. 426, 430.
2 The state of null trade, represented by the 'origin' at which the supply-and-demand curves intersect, is in general a position of unstable equilibrium, that is of minimum advantage; advantage less for both parties than that which is incident to the proximate intersection of the curves, which is in general a position of maximum advantage.
3 *ante*, p. 429.
4 E.g. Mangoldt, Fawcett, Bastable.
5 Compare Ricardo's theory that 'by a continued bounty on the exportation of corn there would be created a tendency to a permanent rise in the price of corn' (McCulloch's edition, p. 188). Compare also the observation made by Mill with respect to taxes considered generally, that a tax, by checking the demand for a commodity, may prevent what we should now call the law of increasing returns from coming into operation (Mill, Book V. ch. 4, § 2, sub finem).
6 Cournot's objection on this score is serious if Mill is held to mean—what he certainly suggests—that England's share of the total gain is in the ratio of (I7 minus...
THEORY OF INTERNATIONAL VALUES

'final' with integral utility; ignoring the principle of 'consumer's rent.' However, it may be admitted that his definition is adequate to the purposes for which it is used. Where he says that the whole or none, or more or less, of the advantage will accrue to a certain country, it is generally true I think, not only in his sense, but in the more correct sense.

The splendid edifice of theory constructed in the first five sections is not improved by the superstructure of later date which forms the latter part of the chapter. This second story does not carry us much higher. What seems at first sight to be an addition will be found, I think, also in the first part; I mean what Cournot calls the 'reflux' of capital and labour; the sort of change which occurs when Germany has obtained cloth from England 'with only seven-eighths of the labour and capital which she previously expended in supplying herself with cloth, and may expend the remainder in increasing her own consumption of linen or any other commodity' (ch. xviii. § 8, first paragraph). But the statement in the original part (§ 5, penultimate paragraph) is equally accurate: 'In the case supposed the consumers of Germany have had part of their incomes set at liberty by the increased cheapness of linen which they may indeed expend in increasing their consumption of that article, but which they may likewise expend in other articles.' (Cf. ibid., last paragraph.)

In short, I agree with Prof. Bastable in regarding the superstructure as 'laborious and confusing.' The last epithet seems...

15) to (20 minus 15); 20 and 15 (yards of linen in exchange for 10 of cloth) being the limits fixed by the respective costs of production, and 17 the value actually set up. (See ch. xviii. ante-penultimate section, et passim.) But Mill need not, I think, be held to that precise statement; and then Cournot's objection amounts to no more than this: that there is a certain asymmetry and inelegance in expressing the share of the total gain in terms of the commodity purchased by one of the parties ('linen').

Cournot's objection is partly directed against the expression of the gain of one party as a percentage—e.g. the gain of England as 20 per cent., if before the trade she obtained 15 of linen, and after the trade 18 for the same quantity of cloth. Has Mill employed such a percentage in the passage quoted in the next note?

1 Cf. Book V. ch. x. § 1, par. 5. 'The amount of national loss is measured by the excess of the price at which the commodity is produced over that at which it could be imported.' Cf. Jevons' Theory, ch. iv., on the gain by exchange.

2 International Trade, p. 29, note.

3 The following interpretation of this difficult supplement may be useful.

We begin by supposing (A) that 'in both countries any given increase of cheapness produces an exactly proportional increase of consumption, or in other words that . . . the [real] cost incurred for the sake of obtaining it is always the same' (sec. 2, par. 2).

A (1). In the first case considered (Ib., par. 5) England expends in procuring linen, whatever its rate of exchange with cloth, the cost of producing a million yards of cloth. Before the trade, England obtained a million yards of linen for
particularly deserved by a certain passage leading to what I have called the second story: where Mill notices the phenomenon of multiple equilibrium, and says: 'It is conceivable that the conditions might be equally satisfied by every numerical rate which could be supposed.' This statement appears somewhat inconsistent with the conception of an equation which Mill has that cost; after the trade, she will not be induced to undergo a greater cost for any amount of linen. The conditions are represented in Fig. 1, where the horizontal $OX$ denotes yards of cloth, the perpendicular $OY$ yards of linen. The cost of producing cloth and linen in England being the same, the indifference curve of England is the line $OA$ making with $OX$ an angle of which the tangent is unity (ante). The demand curve indicated by thick lines is a part of the indifference-curve, viz. $OQ$, and a part of the ordinate through $M$, from $Q$ to infinity; $OM$ representing a million yards of cloth. For at the rate of one of linen to one of cloth any point on $OQ$ may be a point of equilibrium. At that rate England takes 1,000,000 linen in return for the labour-cost by which 1,000,000 cloth are or might be produced; and it is indifferent to England whether she procures that 1,000,000 of linen by producing it all, or by producing any part and obtaining the rest in exchange for cloth, or by obtaining the whole in exchange for cloth. Thus $OQ$ is part of the demand-curve. At the rate of exchange of more than one of linen to one of cloth England is ready to take in return for $OM$ of cloth any amount greater than $MQ$ of linen. Thus $QM$ is the continuation of the demand-curve. By parity $OB$ is the indifference-curve of Germany, $ON$ being $=2OM$. Also $OP$ and $PN$ form the demand-curve of Germany. Accordingly the point of equilibrium is at $P$, on the indifference-curve of Germany; which corresponds to Mill's conclusion.

A (2). In the next case (Ib., par. 4) 'the cloth which Germany had heretofore required was 800,000 yards only, equivalent at the German cost of production to 1,600,000 yards of linen.' This case is represented by Fig. 2 where $ON=1,600,000$; and the demand-curve of Germany is now $OS\ S_n$, while the demand-curve of
elsewhere (Political Economy, Book III, ch. 2, § 3, and review of Thornton, Dissertations, iv.) so well applied to the phenomenon of Supply and Demand. However, suppose that the intersections of the curves are very frequent and close together (as may well be when both are inelastic: ante, p. 430, fig. 4, diagram 4), the case supposed by Mill virtually, if not theoretically, comes into existence. It should be added that Mill has done nothing in his later sections to remove that sort of indeterminateness which does occur in the actual case of plural, though definite, positions of equilibrium—not to speak of that sort of indeterminateness England is as before Q, Qm. These curves intersecting at P, 'the international values will thus be 100 cloth for 160 linen.'

A (3). In the next case it is supposed that 'the million yards of cloth which

![Graph](image-url)

England can make will not satisfy the whole of Germany's pre-existing demand; that demand being (let us suppose) for 1,200,000 yards' (the case put in the note to sec. 7, and by reference to that section included under the first head, our A). This case is represented by Fig. 3, where ON is twice 1,200,000, and accordingly the German demand-curve is O S, S N; while the English demand-curve is as before O Q, Q m. Accordingly the point of intersection being at Q, Germany will purchase a million yards of cloth from England for two million of linen, and will lay out the remainder of the cost, which by hypothesis is constant, in producing for herself 300,000 yards of cloth.

It may be asked, might not Germany, as she gains no advantage by purchasing the million yards of cloth, prefer to produce them herself. The answer to this question, which might equally be asked with respect to case I, is that O, the position of null trade, is not a point of stable equilibrium. Suppose for the moment that all the 1,200,000 yards of cloth were produced in Germany while the linen consumed in England was produced there. It would be for the interest of some of the pro-
which would occur in the case of that *neutral* equilibrium which he imagines.

ducers in Germany and some of those in England to change the direction of their productive forces and exchange German-made linen for English-made cloth on any terms intermediate between one of cloth to one of linen, and one of cloth to two of linen. This process would go on up to the point at which England exports 1,000,000 yards of cloth; in exchange for which the Germans will be forced by competition to give 2,000,000 yards of linen, just gaining no advantage by the trade. Mangoldt's
The chapter on the Distribution of the Precious Metals requires no comment.

In the first section of the Chapter on Competition (Book III. ch. 25), the lenient judgment which Mill expresses appears to explanation of the action of competition in such a case is good. Grundrisse, 2nd edition, Appendix.

I submit that this solution is more correct than that of Mill, who virtually alters the data when he supposes a larger supply of cloth than the by hypothesis constant 1,000,000 yards to be evoked (note to sec. 7). In doing so he abandons the first head which we have called A.

B. We go on now to the class of cases in which the demand is not supposed proportional to the cheapness (sec. 8). This class may be subdivided into two cases:

(1) where 'the proportionality of demand to cheapness holds good in one country, but not in the other, (2) where it 'does not hold good in either country' (loc. cit. par. 3, first sentence).

B (1). The first case under this head, in which 'the demand of England for linen is exactly proportional to the cheapness, but that of Germany for cloth, not proportional,' is represented by Fig. 4, where the German demand-curve is the line of indifference at least up to the point where it meets a perpendicular through the point T, O T = 800,000, as 'she required 800,000 cloth at a cost equivalent to 1,600,000 linen' (loc. cit.). After the point S the demand-curve must leave the straight line as it strikes M in at the point R, M R = 1,400,000.

Another variety of this case is represented by the dotted curve line intersecting O M at point B' M B' = 1,700,000 (sec. 8 par. 2).

The alternative suggested by Mill 'or else tempt England to part with some of the cloth she previously consumed at home' is not proper to case B (1).

B (2). This case is represented by two ordinary demand-curves, Fig. 5, which 'by
imply one at least of the following propositions: (1) The rise of
a competitor may diminish the value without diminishing the
quantity of a country's exports (ibid. last paragraph). (2) A
dimination in the quantity of exports does no great harm to
producers.

The first proposition, I think, cannot be maintained in the
light of the reasoning in Part II, respecting competition. The
second proposition may perhaps be maintained on certain
abstract assumptions. But on the concrete supposition that the
weaker producers of the exported articles may be driven out of
their occupation by a fall in price, and may not be able to find an
equally good occupation elsewhere, the proposition cannot be
maintained.

Mill goes on to argue (ibid., §§2 and 4) that low wages when
common to all branches of industry cannot be one of those causes
which enable one country to undersell another. The argument
is sound if low wages are understood in the Ricardian sense of a
small proportion of the joint product; which is Mill's meaning.
But the argument is not sound, I think, if low wages are under-
stood in the sense of low real remuneration received by
the labourer per unit of produce; oeteris paribus, and in
particular not assuming any elevation in the similarly reckoned
remuneration of the capitalist-employing class—a very natural
meaning to attach to the term. Mill's employment in this con-
exion of the Ricardian dogma that 'general low wages do not
cause low prices, nor high wages high prices within the country
itself' is questionable (§4, par. 9). The Ricardian assumption
that the labour-value of money (the efforts and sacrifices required
to procure a unit of gold) is constant is not very proper to the
mere accident (sec. 8, par. 8) may meet on the line OB making with OM an angle of
which the tangent = 1.6 (the ratio of the total cost of linen to that of cloth) at the
point S, of which the abscissa is OT = 900,000; if England only wants linen to the
amount of nine tenths of 1,600,000 (1,440,000), and Germany only 900,000 of cloth
(loc. cit.).

'In any other case the equation of international demand would require a
different adjustment of international values.' the general case—the comprehension
of which is not much facilitated by the particular suppositions hitherto entertained.
With reference to the interpretation of these sections of Mill, I ought to repeat
that I have had the advantage of reading Professor Marshall's unpublished papers,
referred to with grateful acknowledgment in my Mathematical Part, ante, p. 443.

1 Ante, p. 439.
2 Ante, p. 46.
3 Wages in this sense is, or is proportional to, wages in the sense in which the
term is employed by Mill in the classical passage at the end of his chapter on
Profits (viz., the real remuneration of the labourer per unit of time, loc. cit., par. 2)
divided by 'efficiency' as defined in that section (viz., the amount of work done per
unit of time).
case of International Trade.\textsuperscript{1} It is quite conceivable, if the inhabitants of a country, or a large section of them, are willing to do as much for less remuneration, reckoned in commodities, that the same efforts and sacrifices will procure less gold in the world's market. Accordingly general prices will fall in that country; and in particular the price of exports; thus the country will be able to undersell others where higher wages (in one, and not the least natural, sense of the term) prevail.

In the section on the effects produced on international exchange by duties on exports and imports (Book v. ch. 4) Mill employs a principle which was noticed above as omitted in his first chapter: the converse of the proposition that an improvement in the production of exports may be prejudicial to a country. For when he concludes (loc. cit.\textsuperscript{5} par. 4) that by an export tax in certain cases "England will gain not only the whole amount of the duty but more," is not this "more" attributable to the tax quae impediment? If the tax were intercepted as a transit duty, or otherwise,\textsuperscript{6} this plus would still accrue to the exporting country. The case considered is that which corresponds to Fig. 4 (2) and (4) in our Part II.

The difference above pointed out between the results of a tax on exports and one on imports\textsuperscript{4} may seem not to have altogether escaped Mill. For, while in the case of exports the taxing country may gain "not only the whole amount of the duty, but more," in the case of imports we read (fifth of the paragraphs relating to imports) "taxes on imports are partly paid by foreigners."

In the following section (People's Edition, p. 515b) there is a little inaccuracy. It is not true that "a tax on rare and high-priced wines will fall wholly on the growers, or rather on the owners of the vineyards." If the tax is specific the price will be raised by the monopolist.\textsuperscript{5}

In the section on Protectionism some of the expressions in the 7th paragraph\textsuperscript{6} seem appropriate to the case which I have considered in Part I.: that of a country for whose exports there is an urgent demand in foreign countries benefiting itself by an import tax.\textsuperscript{7}

On the famous passage about "infant industries" I have nothing to add to what has been said by Professor Sidgwick as to the removal of a barrier, so to speak, blocking the initiation of

\textsuperscript{1} Professor J. S. Nicholson, in his masterly article on 'Wages' in the Encyclopaedia Brittanica (vol. xxiv., p. 309c), hints at this exception to the Ricardian principle.
\textsuperscript{2} There is a misprint in the fifth sentence of this paragraph. For "so great" read "as greater."
\textsuperscript{3} Ante, p. 41.  \textsuperscript{4} Ante, p. 495.  \textsuperscript{5} Marshall, Principles, v. 13, 4.
an industry,\(^1\) by Professor Marshall as to the possibility of bringing into play the law of increasing returns,\(^2\) by an ingenioulsy devised system of Protection, and by other eminent economists, in particular Professor Taussig\(^3\) and General Walker.\(^4\)

In conclusion I subscribe to the elevated Utilitarianism which inspires several passages in this section. I trust that Mill has not exaggerated the readiness of the nations to follow an example of commercial disinterestedness—as he has elsewhere certainly exaggerated their readiness to abandon war. ‘Wars,’ says the sanguine philanthropist, ‘are now usually confined, in almost every country, to those distant and outlying possessions at which it comes into contact with savages.’\(^5\) Perhaps ‘collective churlishness’ (Book V. ch. 10, § 1) in commercial relations will die as hard as war.

(3) Cairnes.—Cairnes’ principal contribution to the subject is his recognition of the part played by ‘non-competing groups within a nation.\(^6\) Mill indeed had discerned the existence of such groups;\(^7\) but he made less use of them than might have been expected, even with respect to domestic trade.\(^8\)

Cairnes has also restated the fundamental distinction between foreign and domestic trade at great length and with added clearness; but without I think substantially adding to, or taking from Mill.\(^9\)

On the nicer points of theory Cairnes falls behind his predecessor. He does not seem fully to have apprehended the effect of an improvement in the production of an exported article. In the case of ‘a great improvement . . . in the manufacture of woollen goods in England’ he concludes that ‘English labourers,’ so far as they were consumers of foreign goods procured through an exchange for woollens, would ‘obtain those commodities more cheaply.’\(^10\) This conclusion is erroneous if ‘cheapness’ is defined with reference to some fixed standard, such as labour-cost, for it has been shown that the effect of an improvement in the production of an export might be to make the terms on which

---

\(^1\) *Pol. Econ.*, Book III. ch. 5.  
\(^2\) Address to Section F, British Association, 1890.  
\(^3\) *Tariff History of the United States.*  
\(^4\) *Quart. Jour. of Economics*, April, 1890.  
\(^5\) Book IV. ch. 1, § 2.  
\(^6\) *Leading Principles*, Part III. ch. 2, § 1, p. 386. The subject is well treated by Prof. Bastable in his *Theory of International Trade*, ch. 6.  
\(^7\) As pointed out by Prof. Marshall in his masterly article on Mill’s *Principles of Value, Fortnightly Review*, 1876.  
\(^9\) Compare Prof. Marshall, loc. cit. sub fnem.  
imports are obtained worse.\textsuperscript{1} Cairnes' statements are accurate only on the supposition that alteration in the supply of woollen goods makes no difference in international value. It is only on this interpretation that we can understand his conclusion, 'the wages of English labourers measured in woollen goods would rise in proportion as the cost of those goods had fallen' (p. 407). This is true of a small country, whose influence on international values may be neglected; but is not true in general.

On the important practical question what is the effect of low wages upon the trade of a country, Cairnes is even more open to criticism than Mill. Putting the case of wheat imported into Victoria from South Australia or South America, Cairnes argues 'inasmuch as a rise or fall in the rate of wages [in Australia] has no effect on the comparative quantities of labour required for the production of different commodities, it is evident that if the received theory be true this circumstance must be incapable of altering in any way the course of foreign trade' (p. 390 top, cf. p. 393, par. 2).

Now, as Cairnes fully perceives that comparative cost does not 'determine,' but only 'controls' value (Leading Principles, p. 423), does not fix 'a point about which values move, but a circle within which they move' (ibid. p. 424)—an area corresponding to that intercepted between O T and O S, in our Fig. 6 (p. 623) on the abstract supposition of cost of production not varying with quantity—it might have occurred to him that, even though 'a fall in the rate of wages has no effect on the comparative quantities of labour required for the production of different commodities,' yet, if the Australian workers became disposed to give the same quantity of work in return for less commodities, the point of equilibrium might be displaced to a position such that the Australian goods would become cheaper on the international market. This conclusion does not depend upon the imaginary supposition of fixed costs of production.\textsuperscript{2}

A similar criticism applies to Cairnes' solution of the following problem: 'Suppose a fall of wages to take place in some leading branch of English manufacture—say Sheffield cutlery—... accompanied by a corresponding change over the whole field of English industry... what would be the effect of this on the external trade of England?'

\textsuperscript{1}\textit{Ante} p. 429, where it is shown that the effect of the change might be to push back the position of equilibrium along the supposed unaltered demand and supply curve; that is, to make the gain in respect of utility less for the exporting nation.

\textsuperscript{2} As in the case described, \textit{ante}, p. 46.
The answers given to the problem which is presented by 'supposing the fall in wages not to extend beyond the group of trades in effective competition with the principal industries of Sheffield' (p. 397) seem rather loose from the mathematical point of view. Consider for instance the second of the cases distinguished on p. 397, 'the demand of foreign countries for Sheffield wares' not increased in proportion to their increased cheapness. The answer that there is no answer—'what the exact character of this readjustment would be it is impossible à priori to say'—appears to be inaccurate. The case would seem to be that which is represented by our A B C D E f G H I variety (2) and (4). Accordingly the exporting country will be damnified by the alteration in the terms of trade.

The only defence which can be made is that by a fall of wages Cairnes means only a diminution in the proportion of the national dividend accruing to the wage-earner; not, as it is natural in this connection to understand the term, the diminution in the absolute amount of commodities which the wage-earner obtains per piece. But, as already argued with reference to Mill, this Ricardian definition, however applicable to the case of an isolated country where the labour-cost of money may be assumed to be constant, is less inappropriate to a country affected by international trade, with respect to which the Ricardian proposition, 'high wages do not make high prices' (invoked by Cairnes, p. 390), is deceptive. Cairnes' statement thus defined no doubt is true; but it is misleading in the absence of a more explicit enunciation of that definition.

It will be understood of course that this criticism of details does not touch Cairnes' main contention against popular fallacies on the subject of low wages. The extreme difficulty of our science is illustrated by the reflection that not only are first appearances and common sense—what Cairnes calls 'the com-

1 It is curious that in his Australian and Sheffield examples Cairnes seems to refer principally to that aspect of the problem which may present least practical interest, namely, what would be the effect of a lowered rate of wages upon the country in which they are lowered, abstracting from competition in foreign trade. However, his answer that there is no effect is to be understood as applying to the two more practical questions, (1) what would be the effect on a country dealing with the one in which the wages are lowered; e.g. is America prejudiced by the prevalence of pauper labour in the countries with which she trades? (2) what would be the effect of lowered wages in the country in which they are lowered with respect to foreign competition; e.g. does, or might, England by lowering wages obtain an advantage over America in dealing with a third country?

2 To interpret 'wages' in this connection as day-wages is of course out of the question. This sense belongs to the 'commercial view of the subject' dissipated by Cairnes.
mercial view of the subject—altogether wide of the mark, but even the corrections of the economist require themselves to be corrected. The writer of these criticisms does not flatter himself that they form any exception to this rule.

(4) Professor Sidgwick.—The new theory of international values which Professor Sidgwick has propounded in his Principples of Political Economy, Book II. chap. 3, appears to be tenable upon an assumption which, with respect to modern trade, is plausible, namely that the difference in 'the aggregate of utilities obtainable by similar sacrifices in different localities' (Ibid. § 3, par. 1, 2nd ed.) is not much greater than might be accounted for by the cost of transport. If we assume that any greater difference in the level of advantage would be annihilated by a flow of population (loc. cit.), Professor Sidgwick rightly considers that 'an essential part of the reason why a special theoretic treatment has to be applied to the products of international trade is that a double cost of carriage has here to be taken into account' (Ibid. § 3, par. 2).

The problem which Professor Sidgwick solves might thus be reached, as I understand. First, abstract cost of transport, and let it 'not' be 'assumed that labour and capital do not move freely between the trading countries.' This is the case of ordinary domestic trade. Now introduce a barrier which it requires a certain cost of transport to surmount; Professor Sidgwick applies the general theory of international trade to determine how values would be affected in this particular case.

Putting this or some similar construction on Professor Sidgwick's theory, I accept the positive part of it as true, and perhaps pertinent to a great part of modern trade. But I am unable to accept the negative part of the doctrine, namely that Mill's theory is erroneous, 'unless we further suppose that after the trade is established, there is no product common to the trading countries, a supposition manifestly extravagant' in the case considered (Ibid. § 2, par. 2).

In directing hostile criticism against Professor Sidgwick I feel like a certain attacking party described by Thucydides who, though they had the Lacedemonians at a disadvantage in the island of Sphacteria, yet were oppressed and cowed—literally enslaved—by the prestige of their adversaries. But, like the Athenians on that occasion, I have numbers on my side—not only Mill and all his followers with respect to the general issue, but also at the

1 ἀνιόνες τῇ γράμῳ δεδομένῃ ἃς ἐπὶ Λακεδαιμόνιον' (Thucyd. Book IV. ch. 3, 4).
particular point on which Professor Sidgwick takes his stand, the case of a common commodity, the weighty support of Mangoldt.

Professor Sidgwick argues in the light of a well-chosen example that, if there is a common product, the theory breaks down.

\[\text{For [taking Mill's case of England exchanging cloth for the wine of Spain] let us suppose that there is at least one other commodity—say corn—which is produced both in England and in Spain. According to Mill's general theory of value, discussed in the preceding chapter, the relative values of cloth and corn in England must be determined by their comparative costs of production; and, again, the relative values of wine and corn in Spain must be determined in the same way. But if we suppose cost of carriage to be eliminated, there is no reason why the value either of wine or cloth should be altered by exportation; hence the values of both wine and cloth relatively to corn, and therefore relatively to each other, must be as much determined by cost of production as the values of home commodities are. (Principles, Book II. § 2, 2nd edition, p. 207.)}\]

It appears to me that an injudicious line of attack upon this theory has been adopted by Professor Bastable\(^1\) when he disputes the possibility of there being a product common to both countries—cost of transport having been abstracted—except upon the supposition that the cost of producing the commodities varies with the amount produced. It is quite conceivable that, even on the abstract hypothesis of constant costs of production and no cost of transport, there should be a common product. It is quite legitimate to suppose with Mangoldt,\(^2\) two countries, I. and II. dealing in three commodities, A, B, C; whereof A is produced only in country I., B is produced only in country II., while C is produced in both countries—exported from II., and imported into I. One might even regard this phenomenon as normal, on the plausible hypothesis that there are an indefinite number of articles of trade, with every variety of cost of production.\(^3\) Professor Sidgwick therefore is quite justified in regarding the absence of the phenomenon as "rarely likely to be realized in fact."\(^4\) It is quite open to him to select this ground on which to fight out the issue.

Joining issue with him on the proposition above quoted—

the values of both wine and cloth relatively to corn, and therefore relatively to each other, must be as much determined by cost of production as the values of home commodities are,

I submit that the word 'determine' might here be used in one of two senses: either to mean that value varies proportion-

\(^1\) _International Trade_, Appendix C. " _Hermathena_, 1889.

\(^2\) See the description of his views below, p 633.

\(^3\) Below, p. 634.

\(^4\) _Loc. cit._ 1st edition.
THEORY OF INTERNATIONAL VALUES

ately to cost; or that value varies with, but not in proportion to cost.\(^1\) For example, the first sense is to be understood when Professor Sidgwick, referring on an earlier page of his book to domestic trade, speaks of the Ricardian theory of the determination of value by cost of production;\(^2\) the second sense is to be understood when it is observed by the present writer a few paragraphs below that the international market is determined.\(^3\)

The first sense, according to which the proposition under consideration contradicts the received theory of international value,\(^4\) might have been expected here. But it is expressly disowned by Professor Sidgwick when he says, 'It does not of course follow that the wine and cloth will exchange for each other in proportion to their respective costs.'\(^5\)

In the second sense the proposition under consideration does not contradict the received theory. For it is part of that theory that international values are affected by cost in some way, though not in the same simple way as domestic values. For example, one of the propositions in the fifth section of Mill's classical chapter is that a change in the cost of production of a commodity will in a certain case be attended with a less than proportionate change in its international value. The principal object of our Parts I. and II. is to determine the changes in international value which are consequent upon changes in cost of production; including under cost taxation. In the second sense then the proposition is true; but it does not convict Mill of error. Yet this is the sense in which Professor Sidgwick seems to employ the proposition. But I hesitate to attribute an ignorantio elenchii to the greatest living master of dialectics.

A more certainly valuable contribution to the subject is made in the chapter on Protection; to which our first and second parts are indebted.\(^6\) In this chapter the distinction between the

\(^1\) I have endeavoured to distinguish the two meanings in the article on Exchange Value in Palgrave's Dictionary of Political Economy. The distinction is quite clearly indicated by Mill (Pol. Eco., Book III. ch. 18, § 9 and § 5).


\(^3\) Below, p. 699, par. 5.

\(^4\) It may be observed that the supposed product common to both countries, far from evidencing the truth of the proposition under consideration—as the turn of Prof. Sidgwick's sentence might suggest—is properly employed by Mangoldt as the very type and measure of that difference in the productivity of the two countries from which follows the truth of the received theory, the falsity of the proposition in the first sense. See the example cited below (p. 692), where the (real) costs of producing C, the common product in the respective countries, are in the ratio 8 : 4.


\(^6\) Ante, pp. 49, 499.

No. 16.—Vol. IV. 
good of one country and of all (§ 1); the proof that a country may by an import tax benefit itself in the way of revenue while it protects native industries (§ 2), and that a large section of a community may be injured by free trade (§ 3) appear especially masterly.

(5) Professor C. F. Bastable.—Professor Bastable's 'attempt to restate, in a more complete form, the doctrines of the classical English school' 

on International trade, has been attended with a large measure of success. The classical or Ricardian method admits of completion on two opposite sides; two contrasted deficiencies may be attributed to it. On the one hand, it appears not to go to the root of matters. It is as if an astronomer should content himself with the proposition, 'Planets move in ellipses,' without ascending to the first principles of dynamics. Such a one might be at a loss when he comes to deal with certain comets. On the other hand, the proposition that planets move in ellipses might be a useless abstraction in a conceivable cosmos where the existence of a disturbing medium caused the theory to lag very much behind the fact.

It appears to me that Professor Bastable's completion of the classical method in the latter direction is quite complete. He has fully learnt the lessons of caution which have been taught by his great countrymen, Leslie and Ingram. He never forgets that the 'hypotheses' of economics 'are unreal, or at all events incomplete.' He has avoided the more dangerous extreme, the Charybdis of over-abstraction.

But I am not so sure that he has kept clear of Scylla; and I shall attempt to indicate some instances in which deduction from first principles would lead to conclusions different from his.

It is a little misleading to compare the trade between two nations, supposed to be the only two in existence, to the 'terms of an exchange between isolated individuals.' The suggestion that the terms are indeterminate in the former case in the same sense as the latter appears to be theoretically indefensible. The usual assumptions being made that there is a large number of competing dealers on each side, the rate of exchange is to be regarded as determinate in the international market as well as in the home market. Accordingly, the analogy of monopoly

1 International Trade, Preface.
2 Some of the following sentences are taken from a review of Prof. Bastable's work by the present writer, which appeared in the Academy for May 21, 1887.
and combination should, I think, be sparingly applied to international trade.¹

The introduction of the idea of monopoly adds difficulty to a passage in the chapter on the 'Theory of International Values,' which in the absence of mathematical representation I am unable to interpret with confidence. The writer appears to suppose that the part played by utility would be different if, to use Mill's familiar example, the comparative cost of production in England were not 10 of cloth to 15 of linen, but 10 of cloth to less than 10; e.g., 1 of linen. I cannot regard this distinction as essential. In both cases the curve of indifference is represented by a straight line, the costs of production being supposed constant. Thus OT in the figure, the tangent of TOX being 2, represents those states of trade in which Germany would be no better off than if there had been no foreign trade; the cost of producing linen comparatively with that of producing cloth in Germany

¹ The use of the term monopoly in a sense not involving the attribute indeterminateness is allowable (e.g. International Trade, p. 115, note. Cf. ante, p. 43, par. 1). It is tenable, too, that transactions between two countries, though determinate, are less steady than where these are competing nations.

² p. 37.
being as 10 to 20. Similarly OS in the figure, the tangent of SOX being \( \frac{1}{10} \), and the cost of the production of linen comparatively with that of cloth in England being as 10 to 1, represents those states of trade in which England would be no better off than if there were no foreign trade. The determination of the point of equilibrium involves what may be called 'the comparative utility of the commodities \( x \) and \( y \) to the consumers in \( B \)' [our 'England']; in the same sense, I think, whether \( B \) is able to produce \( y \) at the amount of 15 per unit of productive power . . . [15 of linen at the same cost of 10 of cloth] or can only win from its own resources a very small amount, say 1 \( y \) for each unit' [1 of linen at the same cost as 10 of cloth].

I do not understand the difficulty raised by Cournot\(^1\) to which Professor Bastable makes reply in the passage just referred to in quite the same sense as Professor Bastable. Cournot's difficulty is only, I think, that which Professor Bastable considers at his p. 44: the difficulty of understanding Mill's rule for the division of the gain by trade. As I have already pointed out,\(^2\) Cournot hits an inaccuracy on a very plausible interpretation of Mill; on any interpretation, an inelegancy. I do not understand Professor Bastable's reply at the latter passage\(^3\) to Cournot.

The relation of import to export duties is another point with respect to which the Ricardian and the mathematical methods lead to somewhat different results. The 'kind of symmetry,' the 'parallelism,' which Professor Bastable predicates in his paper on The Incidents and Effects of Import and Export Duties, is not in conformity with the propositions which I have attempted to prove.\(^4\) The symmetry in the action of the two kinds of taxes leads, according to Professor Bastable, to a marked discrepancy in their results. For 'the essential point of difference is that the export duty affects a limited area of production, the import one a limited area of consumption; '\(^5\) and since, as a matter of fact, the area of production is much more frequently limited than the area of consumption,\(^6\) 'it would therefore appear that it is almost impossible to tax foreigners by the instrumentality of duties on imports.' This conclusion is at variance with that which has been above deduced from first principles. Provided that the area of pro-

\(^1\) Principe de la th\'eorie des richesse, 1868, pp. 344, 345.
\(^2\) Above, p. 609.
\(^3\) International Trade, p. 44, note.
\(^4\) See ante, pp. 234, 435.
\(^5\) Incidents and Effects, p. 4, note.
\(^6\) International Trade, p. 114 and context.
duction is limited, it is not so much matter which instrumentality is adopted.

B. We come now to the category of foreigners, which, as already observed, is nearly coincident with that of mathematicians.

(1) Cournot.—The lesson of caution in dealing with a subject and method so difficult is taught by no example more impressively than by that of Cournot. This superior intelligence, equipped with the most scientific apparatus, seems not only to have slipped at several steps, but even to have taken a wholly wrong direction. He has not only committed errors in formal reasoning, but also has missed general conceptions appropriate to the subject.

Of several paradoxes which occur in that part of the Principes Mathématiques which more immediately relates to International Trade, perhaps the first is among the few that are not open to suspicion. This is the proposition that, when a communication is opened between two markets, previously separated by a barrier, the total quantity produced of any commodity which now begins to be exported from one market and imported to the other will not necessarily be increased. For if a flow sets in from market A to market B, the production of the commodity in A must be increased, and its price in that market heightened—the law of decreasing returns prevailing; while in B the price will be lowered, and the quantity produced in that country will be diminished. The increase of the production in A may not compensate the decrease in B; when the demand in A is very inelastic, and the rise in the cost of production with the amount produced very steep, while the contrary properties are true of B (Art. 68).

A similar proposition is true of the total value of the product (Art. 69).

The conditions under which these propositions are true are well expressed by Cournot’s symbols, in which \( \Omega_A(p) \) = the amount offered by the producers in A at the price \( p \), and \( F_A(p) \) means the amount demanded by the consumers in A; with similar interpretations of \( \Omega_B(p), F_B(p) \). Thus, before the communication,

\[ \Omega_A(p_0) = F_A(p_0); \]

\( p_0 \) being the price of the article in the market A; and, after the communication, if the commodity is exported from A to B, \( \epsilon \) being

1 Ante, p. 46.
2 Recherches sur les Principes Mathématiques de la théorie des richesses (1838), ch. x. xi. xii.
the expense of transportation per unit of commodity, and the price in A being changed from \( p_a \) to \( p_a + \delta \), we have

\[
\Omega_a(p_a + \delta) + \Omega_b(p_a + \delta + \epsilon) = F_a(p_a + \delta) + F_b(p_a + \delta + \epsilon)
\]

(Arts. 67 and 68.)

We have now to enquire whether the quantity denoted by either member of this equation is greater than the corresponding quantity before the communication was opened; whether the following inequality holds:

\[
F_a(p_a + \delta) + F_b(p_a + \delta + \epsilon) > F_a(p_a) + F_b(p_a).
\]

Cournot answers this question in the negative by showing that the inequality does not hold in a particular case: namely, when the original prices, \( p_a \) \( p_b \), differ from each other, and also from the new price in A, by only a small quantity, in which case also the cost of transport, \( \epsilon \), must be small, since otherwise exportation from A to B would not take place on the removal of the barrier. This reasoning, or that which is based on another particular assumption, viz. \( \delta \) and \( p_b - (p_a + \epsilon) \) small (Art. 68, last par.), is quite correct. But the assumption that \( \epsilon \) should be small leads to an erroneous conclusion in a subsequent problem: to determine the effect of a tax on exports or imports (Art. 70).

If \( p \) is the price of the article in the exporting country before the imposition of the tax \( u \), and \( p + \delta \) the price after the tax, we have, before the tax, \( \Omega_a(p) + \Omega_b(p + \epsilon) = F_a(p) + F_b(p + \epsilon) \). And after the tax \( u \) per unit of commodity has been imposed, we have

\[
\Omega_a(p + \delta) + \Omega_b(p + \delta + \epsilon + u) = F_a(p + \delta) + F_b(p + \delta + \epsilon + u).
\]

Cournot now proceeds to draw conclusions from the last equation by expanding and neglecting the powers, not only of \( \delta \) and \( u \), but also \( \epsilon \), above the first power. I submit that Cournot's

1 For \( p_a + \delta \) being the price of the commodity in A, and accordingly the (net) price which the producers in A obtain (not only for that portion of the product which they sell in A, but also) for that portion of their product which they sell in B at a price heightened by the cost of transport \( \epsilon \), the quantity offered by the producers resident in A at the (net) price \( p_a + \delta \), together with the quantity offered by the residents in B at the price \( p_a + \delta + \epsilon \), is equal to the quantity demanded by the residents in A at the price \( p_a + \delta \), together with the quantity demanded by the residents in B at the price \( p_a + \delta + \epsilon \).

2 Compare the last note.
procedure is inelegant and leads him to an erroneous conclusion. The simpler procedure is first to treat $\delta$ and $u$ only as small, $\delta$ being the dependent, $u$ the independent variable. Thus,

$$
\delta(\Omega'(p) + \Omega'(p + \epsilon) - F'(p) - F'(p + \epsilon)) \\
= -u(\Omega'(p + \epsilon) - F'(p + \epsilon)).
$$

If now $\epsilon$ be small, we may expand both sides of this equation in powers of $\epsilon$, and neglect terms involving powers of $\epsilon$ above the first, or rather neglect $\epsilon$ altogether. Whether $\epsilon$ be small or not, it follows—the law of diminishing returns, as well as that of diminishing utility, prevailing—that $\delta$ is negative, and less than $u$; or that the price falls in the exporting country and rises in the importing one, contrary to the statement of Cournot (§21, par. 1).

I am confirmed in this view by Mr. A. Berry and Mr. C. P. Sanger, who have independently made a similar correction. Mr. Berry writes to me of the corrected reasoning: 'This may be confirmed by the fact a priori evident that the disturbance of price, $\delta$, must vanish when the tax itself, $u$, vanishes. This is the case in our equation, not in Cournot's.'

It is certainly curious to find a wrong belief as to a matter of fact in business resulting from a slip in mathematical analysis!

Mr. Berry has pointed out to me another slip in Art. 90, pp. 183, 184. There a certain advantage which the author ascribes to domestic as compared with foreign trade does not follow from his own premises.

To this I have to add that those premises are very doubtful. I allude to the theory of 'real' as distinguished from 'nominal' revenue. To collate here all the passages in all Cournot's versions which bear on this distinction would occupy too much space. It must suffice to submit as the result of such an examination very carefully performed the opinion that, while Cournot's 'nominal revenue' is much the same as what would now be called the money measure of national wealth, his 'real revenue' signifies, if indeed it is significant, such a measure as that which Mr. Giffen, Mr. Bourne, and others have employed in determining the growth of the quantity of a nation's 'capital,' or foreign trade. Such a measure is obtained by multiplying the quantities of each commodity at the two compared epochs by its price at one of them, the same price being combined with the two quantities, the one at the initial and the one at the final epoch. Consistently with this view Cournot says that if the price of a commodity rises from $p_n$ to $p_0$, corresponding to a diminution of the quantity from
D₀ to D₁, whereas the variation of the nominal revenue is
D₀p₀ — D₁p₁, the loss in real revenue is (D₀ — D₁)p₀.

I do not indeed pretend to follow the double route by which
Cournot, winding his way through additions and subtractions of
producers' and consumers' gain and loss, reaches this conclusion
(Principes Mathématiques, ch. xi., and corresponding passages in
the Principes of 1863 and the Revue Sommaire). Nor can I
explain why, upon the interpretation of real revenue here suggested,
the loss due to a rise of price should be formulated as (D₀ — D₁),
multiplied by p₀ rather than p₁; except so far as in the method in
question there must be always something arbitrary in the selection
of the price to be operated with.

However the conception of 'real revenue' may be interpreted,
it does not seem appropriate to the problems in hand. According
to Cournot the real revenue of a country is diminished by the
admission of an additional import through the removal of a
restriction on trade. The capital objection to this conclusion is
that no account is taken of that sort of advantage coming from
cheapness which we should now describe as Consumer's Rent.
Cournot explicitly makes abstraction of this advantage. He says
of it—

Dans l'évaluation de l'accroissement réel du revenu social, causé par la baisse de
prix, on ne tient pas compte de l'avantage qui consiste, pour les nouveaux con-
sommateurs de la denrée, à faire un emploi plus à leur goût d'une portion de leurs
revenus; parce que cet avantage n'est pas numériquement appréciable.' (Art. 81.)

Of the corresponding loss he says:—

' Il s'agit ici d'un de ces rapports d'ordre, et non pas de grandeur, que les nom-
bres peuvent bien indiquer, mais non pas mesurer . . . nos considérations ne portent
que sur les choses mesurables. (Art. 77.)

Ce dommage n'est pas mesurable et n'affecte pas directement la richesse nation-
ale, dans l'acceptation commerciale et mathématique de ce mot.' (Art. 88.)

1 Prof. Seligman seems to follow Cournot without hesitation. He puts the follow-
ing case (Shifting and Incidence of Taxation, p. 153): 'Suppose that the price
of the commodity was originally $10, at which price 10,000 pieces were sold. Now a
tax of $2 is imposed, all of which is shifted to the consumer. At the new price,
however, only 8,000 pieces will be sold.' Manipulating the producers' and con-
sumers' loss in Cournot's fashion, Prof. Seligman reaches the conclusion that 'the
diminution in the real revenue = $20,000.'

As it seems to me, the essential fact is that there has been a diminution of the
national wealth to the extent of 2,000 pieces of the taxed commodity. It is arbitrary
whether we multiply this 2,000 by 10, the old price, or 12, the new price, with a
view of ascertaining (after the manner of Mr. Giffen) the variation in the total
quantity of national wealth, provided that, in dealing with other items of national
wealth at the two periods, we employ the corresponding prices—either the old
prices or the new. Perhaps the best price to operate with would be a mean of the
old and new price, in the case before us $11.
Real revenue being thus defined, the proposition that it is diminished by the liberation of trade may be true, but is not important; as Bertrand urges in an interesting criticism on mathematical economists.\(^1\)

Another objection to Cournot’s proposition raised by Prof. Bastable is that it uses money as a measure; whereas the value of money is altered by an alteration in the terms of international trade. It is tenable, however, that Cournot means to restrict his theory to small disturbances of trade, the effect of which on the level of money may be neglected. As far as this objection goes, his reasoning may be as valid as Prof. Marshall’s application of Consumer’s Rent,\(^2\) or Messrs. Auspitz and Lieben’s reasoning as to the effects of a tax or bounty.\(^3\)

Another objection to Cournot’s reasoning is that he does not take account of the productive factors which, being displaced by the importation of a commodity which had been produced at home, are turned to the production of some other commodity. Cournot himself has stated this objection, and endeavoured to meet it (Arts. 93 and 86); but I do not feel certain that on this point he gets the better of Hagen, to whom we now proceed.

(2) \textit{Hagen}.\(^4\)—The mathematical method is not wielded by Hagen more powerfully in defence of Free Trade than by Cournot against it. Hagen constructs an ‘exportation-formula’ to represent the gain (or loss) resulting to the national income from a new export (p. 11). This gain consists of three parts: (1) the addition to profits consequent upon the additional production of the exported article; (2) the loss of profits consequent upon the transference of productive factors from other industries to the production of the exported article; (3) the loss to consumers consequent upon the rise of price. This formula appears open to three serious objections: (a) It is assumed that profits in different industries at the same time are a fixed proportion of the expenses of production. This Ricardian assumption may perhaps pass. But not so (b) the ultra-Ricardian neglect of all interests but those of the capitalist; no account being taken, as I understand, of the effect of the supposed change upon wages and rent. Lastly (c), the effect on the consumers’ interest is not

\(^{1}\) \textit{Journal des Savants}, 1883.


\(^{3}\) \textit{Cf.} below, p. 638.

\(^{4}\) \textit{Die Notwendigkeit der Handelsfreiheit für das Nationaleinkommen Mathematisch nachgewiesen}, Von Karl Heinrich Hagen, Königsberg, 1844. \textit{See article on Hagen in Palgrave’s Dictionary.}
rightly formulated. The price being raised from $P$ to $P + p_1$, and the amount consumed being diminished from $D$ to $D - d$, Hagen puts for the loss of the consumers $p$ ($D - d$). If he had added $\frac{1}{2} p \times d$, this would have been an intelligible measure of the loss of consumers' rent; being, in fact, the expression which Dupuit—with as much accuracy perhaps as the subject admits of—has put for what is now called consumers' rent.\(^1\)

From this formula Hagen concludes that export trade may or may not be disadvantageous (p. 14). By parity of reasoning he finds that importation must always be advantageous (p. 16). A small bounty may be attended with a slight gain. It may be questioned whether, in view of the unsoundness of the premises, any value attaches to these deductions.

In conclusion, Hagen joins issue with Cournot on two points corresponding to the second and third term of Hagen's exportation-formula (above). On the question whether the productive factors which are displaced by exportation or importation should be taken into account, Hagen seems to have the better of Cournot.\(^2\) In the matter of consumers' rent it is not easy to say which is most in the wrong, Cournot who ignores, or Hagen who falsifies the theory. Indeed, a similar difficulty affects the comparison between the two authors' whole treatment of International Trade.

(3) Mangoldt.\(^3\)—This author leads up to the subject of International Trade by some sections on Exchange (§§ 62—74, 1st edition), in which he represents Demand and Supply by curves very similar to those which are now in vogue. In virtue of these constructions Mangoldt, writing without reference to his predecessors, Cournot, Dupuit, and Gossen, may claim to be one of the independent discoverers of the mathematical theory of Demand and Supply.

In his Appendix (Anmerkung) On the Equation of International Trade Mangoldt begins by following Mill's supplementary sections,\(^4\) dividing the subject according as the demand for a commodity is, or is not, inversely proportional to its price. Under the first head Mangoldt considers first the case of two variables, and deduces conclusions substantially identical with those of Mill, in usefully varied language. Mangoldt then goes on to the

---

\(^1\) See article on Dupuit in Palgrave's Dictionary.

\(^2\) Cournot has replied in his Principes of 1863, Art. 185. Hagen speaks of reviewing Cournot's work as a whole. Does such a review exist?


\(^4\) Above, p. 608 note.
case of three or more variables. He discerns the general proposition that—cost of production being supposed constant irrespective of quantity, and abstraction being made of cost of transport—if trade is opened between two countries, the commodities previously produced in both countries will now fall into two groups, each produced altogether in one country: the rate of exchange between the members of each group inter se corresponding to the cost of production of each commodity (in the country in which it continues to be produced), and the relation between the two groups being determined by the rate of exchange between the produce of a unit of productive force in one country and that of a similarly defined unit in the other country.¹

This simple truth Mangoldt complicates by positing a commodity as it were intermediate between the two groups, which may serve as a measure whereby to ascertain from which of the countries any particular commodity will be exported.

The following construction of our own seems to give the substance of Mangoldt's expositions; it being understood that the substance, as the metaphysicians say, is not a copy of its manifestations. Let us figure the relation between the costs of production of the set of commodities in Country No. I. by a series of points a, b, c, &c., on a right line, any one of which a is obtained by measuring from a fixed origin o, a distance equal to the logarithm of the number of units of productive force which go to the production of a unit of that commodity in Country No. I. Let the natural values of the commodities in Country No. II. be similarly designated by the points a', b', c', &c., measured from o'; o' being taken so that oo' is the logarithm of the number of units of productive force in Country No. II. of which the produce is equivalent in the international market to the produce of a unit of productive force in Country No. I. (log v, or log $\frac{1}{v}$ in our notation²). It appears at once from the figure that, when trade has been established, it is cheaper for Country No. I. to import a', b', and c' than to produce them; and to produce d and e than to export them.

The measure or standard which Mangoldt desiderates would be afforded by the commodity, if the distance between c

¹ The v of our formula ante.
² Ante, p. 441.
and c vanished. That commodity would be on the line between imports and exports; and it would in general be partly produced and partly imported by one and the same country. Mangoldt illustrates this conception by the following example. Let the costs of production of the three commodities A, B, C be in the first country 2, 3, 4 respectively, and in the second country 4, 2, 3 respectively, as shown in the annexed scheme.

\[
\begin{array}{ccc}
\text{A} & \text{B} & \text{C} \\
\text{I.} & 2 & 3 & 4 \\
\text{II.} & 4 & 2 & 3 \\
\end{array}
\]

And let the amounts demanded by each Country before the opening of the trade be as follows:

\[
\begin{array}{ccc}
\text{A} & \text{B} & \text{C} \\
\text{I.} & 1,000 & 800 & 600 \\
\text{II.} & 500 & 750 & 600 \\
\end{array}
\]

Then by hypothesis (according to the definition of the first class of cases ²) country No. I. lays out a constant cost of \(1,000 \times 2 = 2,000\) units of her productive force—in procuring commodity A for her own consumption, 800 on B; and so on. Employing this datum, by a tentative process, Mangoldt reaches the conclusion that A will be produced in No. I. only, B will be produced in No. II. only, C will be produced both in No. I. and No. II. Of A there will be produced in No. I. for her own consumption 1,000, for export \(133\frac{1}{3}\). Of B there will be produced in No. II. 750 for her own consumption, 900 for export. Of C there will be produced in No. I. \(533\frac{1}{3}\) for her own consumption, and there will be imported \(66\frac{2}{3}\); and in No. II. there will be produced 600 for her own consumption, and there will be exported \(66\frac{2}{3}\). The new values are:

\[
\begin{array}{ccc}
A & B & C \\
2 & 2\frac{1}{3} & 4 \\
\end{array}
\]

Here C occupies an intermediate position between exports and imports, as may be verified by remarking that, after the trade has been set up, neither country can gain by either exporting or importing C. For it costs 4 units of productive force in No. I., and 3 in No. II.; and the produce of 4 units of No. I. is equivalent on the international market to the produce of 3 units of No. II., as appears from the fact that after the trade has been opened, A and

---

² Above p. 630, and cf. p. 609 note.
B, each the product of two units in the country in which it continues to be produced, are valued at 2 and $2\frac{3}{4}$ respectively, or in other words exchange at the rate of 8A for 6B.

This theory brings into view an incident which is apt to be masked as long as we confine ourselves to the case of two commodities, the classical 'cloth' and 'linen'—namely, that it is not in general possible to determine \textit{a priori}, from a mere observation of the costs of production in the respective countries before the opening of the trade, which commodities will be imported and which produced at home. 'Comparative cost' cannot be ascertained by simply comparing the costs of different articles in the two countries. Thus if o' in the figure be pushed up a little, the distances o' a', o' b', &c., being preserved constant, C will become an export (from country No. I) instead of an import. But the position of o' depends not only on the cost of production in each country, but also on the law of demand in each country for the different commodities.

This incident is illustrated by one of Mangoldt's examples, in which the costs of production of five commodities in the two countries before the trade may be thus represented (p. 218)—

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>I.</td>
<td>4</td>
<td>7</td>
<td>6</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>II.</td>
<td>5</td>
<td>9</td>
<td>3</td>
<td>7</td>
<td>4</td>
</tr>
</tbody>
</table>

Upon a certain hypothesis as to the amount of each commodity demanded by each country (it being recollected that the real cost laid out on each article by each country is supposed to be constant), it is found that A and B are produced only by No. I., C and E only by No. IV., while D—"the measure of the relative productivity of the two countries"—is produced in both. But if the quantities demanded were different, D would be produced only in No. I. (pp. 220–222). From the examples in the textbooks it might have been supposed that D would necessarily have been exported from the second country, and E from the first; since thus the second country could get its E cheaper—namely, at a rate less than $\frac{3}{4}$ D for one of E; and the first country could get its D cheaper—namely, at a rate less than $\frac{3}{4}$ E for one of D. But the truth is that in general no conclusion of the kind can be drawn pending the determination of the relation on the international market between the productive powers of the two countries, the ratio which we have designated as v. It is as the material embodiment of this relation between quantities of
labour and sacrifice that Mangoldt’s conception of a standard commodity is significant.

But an actual commodity subserving this purpose is not always to be found, as appears from the example which we have just cited, and as Mangoldt himself has pointed out. It may be observed that an actual standard would be forthcoming on one hypothesis—namely, that the volume of trade is split up into an indefinitely large number of items with every variety of cost of production; but in this case the standard commodity, though existent in fact, would probably be insignificant in magnitude.

The results of the abstract problem with which the investigation started are summed up at p. 223 in a set of italicised propositions, which may be read with assent and instruction. The first alone excites some scruple:

‘There come first into international trade those commodities of which the costs of production compared with the costs of production of other commodities in the same land differ most widely from each other, then those for which the difference is next greatest.’

At first sight there seems to be contained here a statement as to the path or process by which the position of equilibrium is reached; whereas the equations of exchange enable us at best to determine the final position, not the steps by which it is reached. What Jevons called the ‘Mechanics of Industry’ is statical, not dynamical. It appears, however, from the context that the author is aware of this characteristic. The assertion which he makes in the proposition cited relates only to the first step—not to the intermediate path—towards equilibrium; and the affirmation that the first step taken will be the most advantageous one to both parties is tenable.

The simplest case having been discussed, Mangoldt proceeds to restore certain attributes which he began by abstracting.

First let us no longer suppose the quantity demanded to be in inverse proportion to the labour-cost, but to vary with the rate of exchange between exports and imports, according to some more complicated law. The law which Mangoldt specially affects is such that when the rate of exchange or ‘price,’ P, is changed to Pm, m being any factor, the quantity demanded, N, becomes

1 I have had occasion to defend this view against Professor Walras in the Revue d’Économie Politique for January 1901.

2 ‘Die Art und Weise wie sich der process der Vertauschung der Production vollzieht ist an sich gleichgültig’ (p. 213), [das] ‘das Endergebniss immer das nämliche bleiben wird’ (p. 216, last par.)
THEORY OF INTERNATIONAL VALUES

635

\( r \times \frac{1}{m} N \); where \( r \) is an improper fraction, in cases instanced by

the author, \( \frac{1}{2} \) and \( \beta. \)

Employing this conception, Mangoldt enunciates that condition of equilibrium which would now be
described as the intersection of two curves.

He then goes on to consider the phenomenon which would
now be described as the multiple intersection of demand and
supply curves (pp. 228, 229, and cf. § 68). His views on this
curious subject are very interesting. He thinks that in general
of several possible positions of equilibrium that one tends to
be realised which is most favourable to the more active
of the two nations. But there are stated some probabilities on
the other side, which seem not very easy to apprehend (p.
229). It may be observed that Mangoldt, like Mill, supposes
neutral equilibrium—the coincidence of the two curves as we
may say—to be possible.

So far the cost of production has been assumed to be con-
stant, whatever the amount produced. Mangoldt next supposes
(p. 232) the relation between cost and quantity which is now
called the law of diminishing returns to prevail, and illustrates
the general theory by a particular example, which is rendered
more workable by resorting to the simple law of demand at first
assumed—namely, that the quantity demanded is in inverse
ratio to the cost.

Finally, the cost of transport is taken into consideration
(p. 233). Mangoldt propounds the remarkable theory that upon
a certain hypothesis the carrying trade between two countries
tends to fall to that one which has the smaller absolute produc-
tivity (p. 235). The distinction between the 'active' and 'passive'
nation which we have already met with in connection with
plural equilibrium here recurs (p. 240). Mangoldt illustrates his
theories more suo by laborious examples. He sums up the section
on cost of transport in a series of propositions, among which the
following—very freely paraphrased—seem the most remarkable.

(1) The carrying trade between two nations tends to fall into

---

1 As I understand, if (as in Cournot's demand curve) \( x \) be the price and \( y \) the

corresponding quantity demanded, \( = f (x) \); we have \( f (m x) = \frac{r}{m} f (x) \).

In the particular case where the law applies only to small changes of \( x \), put

\( m = 1 + a \), a small. Whence \( y + a \frac{dy}{dx} = y - a r y \).

\[
\frac{dy}{y dx} = - r. \quad y = Ce^{-rx}
\]

2 Above, p. 610.
the hands of one, a tendency counteracted by what, with reference to abstract theory, may be described as accidental circumstances.

(2) The carrying trade tends to fall into the hands of that nation the volume and weight of whose exports are greatest.

(3) An improvement in productivity tends to deprive a country of a share in the carrying trade.

(4) Improvements in means of production redound in general, and in the abstract, to the good of the importing people only.

These propositions appear to be, not indeed incorrect—as defined and qualified in the context—yet unimportant. Considering, however, the solidity of the rest of Mangoldt’s work, it may well be that one specially interested in the problem of the apportionment of the carrying trade would discern more in this last section than the present writer, after taking a reasonable amount of trouble, has been able to find.

(4) *Auszips und Lieben.*—In that portion of the *Théorie des Preises* which treats of international trade, the subject is enriched with important propositions and embellished with splendid illustrations. Perhaps the most valuable result due to the authors is the general geometrical proof that a nation may benefit itself in certain cases by an import or export tax. The construction by the aid of which they have discerned this theorem more clearly than their predecessors is much the same as that which has been employed in the earlier pages of our mathematical part: down to the introduction of complicated curves corresponding to organic changes in trade. But there is one important difference between even our simpler constructions and theirs: that theirs are restricted to a small part, ours are applicable to the whole volume of trade. Their abscissa represents a real article, one out of the many items in international trade; their ordinate represents money, the marginal utility of which is properly considered as not varying with the amount consumed of a single article. Each of our co-ordinates on the contrary represents not so much actual commodities or money, as an ideal article typical of the total volume of trade; used to suggest conclusions which may be verified by the algebraic analysis proper to the real case of numerous exports or imports. Accordingly their supply- or offer- curve is never *inelastic* in our sense of the term; it continually ascends like the curve O E in the annexed figure; since, if money have a constant utility-value, for a higher price

---

1 *Théorie des Preises*, fig. 74.
2 *Ante*, pp. 424, 442.
3 *Ante*, pp. 426-435.
4 *Ante*, pp. 428, 429.
more (or not less) of a product (subject to the law of decreasing returns) will continually be offered. For a converse reason our curve may curl round like the dotted line in the figure. In short, the varieties of curve marked as (3) and (4) in the fourth figure of our Mathematical Part,¹ do not occur in their scheme. Accordingly they are not conducted to a certain proposition which we have typified by the statement that, if Europe had an urgent demand for the produce of the United States, it might be for the interest of the United States to put an import tax on the produce of Europe. Now as long as we consider the supply curve for European articles as of the form O E, an import tax thereon cannot come to much, as the authors observe (Theorie des Preises, p. 417). The curling round of the curve is required to express the urgency of the European demand for American produce. While we consider the supply curves of particular articles of the form O E, we do not get beyond the effect which we have likened to the buffer of a railway carriage being pushed back;² to contemplate the movement imparted to the whole train, we

¹ Ante, p. 430.
² Ante, p. 46.
require a construction such as that which has been employed by us.

Another difference between our and their constructions is that they seem to confine themselves to the simpler species of curve which we have called primary (ante, p. 430). With reference to the law of supply and demand thus conceived, they rightly argue that a bounty can never be beneficial to the community as a whole (Theorie, p. 425). They miss Professor Marshall’s conclusion that a bounty attended with what we have called organic changes, bringing the law of increasing returns into play, may be beneficial.\footnote{Ante, p. 438.}

I trust that this third and concluding part of my study on international value will corroborate the two preceding parts: that the theories enounced in those parts will be at once confirmed by their general agreement, and not discredited by their occasional discrepancy with the principal authorities on the subject. I regret that the negative portion of this result could not be attained without the use of controversy.

F. Y. Edgeworth